RELIABILITY AND AVAILABILITY EVALUATION OF A SYSTEM SWITCHED TO ANOTHER SIMILAR, SUBSTITUTE OR DUPLICATE SYSTEM ON TOTAL FAILURE

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Abstract: A two-unit standby system is considered with two types of repair facilities. One facility repairs one unit at a time and other facility repairs both the units simultaneously. When both the units fail, if unit can be repair in short time then repair will be continued, otherwise in order to improve availability another substitute system taken from outside is used, which is guaranteed for failure free operation. Assuming failure and repair times as exponentially distributed, Expressions for the mean time to system failure (MTSF), the steady state availability and busy period for system are derived using linear first order differential equations. A particular case for the proposed system is discussed in which substitute system was not considered. Also comparison is performed graphically to observe the effect of the proposed system on Availability.

Key Words: Availability, Linear first order differential equation, Mean Time to System Failure, Reliability, Steady State Availability.

1. INTRODUCTION:

Competition exists in every field, to keep ahead a major challenge is availability improvement of a system, as less availability has negative impact. People often use "availability" and "reliability" interchangeably. In fact, however, the two terms are related but have distinct meanings. Reliability (as measure of the mean time between system failures, or MTBF) is one of two key components of availability. The other is the mean time required to repair a given system when it fails, or MTTR. The formula for availability is as follows:

Availability = MTBF / (MTBF + MTTR)

We can have example of a power supply system which is highly reliable, because it rarely experiences downtime, but not highly available because it has a high mean time to repair.

Two-unit standby systems are studied by several authors including Mohammad El-Moniem Soleha who studied Reliability and Availability characteristics of a two-dissimilar-unit cold standby system with three modes by using linear first order differential equations. M.Y. Heggag (2009) analyzed cost of two-dissimilar-unit cold standby system with three modes and preventive maintenance by using linear first order differential equations, Upasana et.al. (2011) analyzed a two unit standby oil delivering system with a provision of switching over to another system. L.R. Goel et.al. (1983) analyzed two unit cold standby system with two types of repair and preventive maintenance. Gupta and Sammerwar (2000) studied a standby system with varying rates of failure, repair, inspection and post repair. Gupta and Bedre (2005) analyzed a fault tolerant system with two types of failure. But very less attention was paid to improve availability. The present study is an effort to improve availability, here the system consists of a single unit with a cold standby unit with different failure rate. When original unit fails, standby unit becomes operative. There may be a situation when both the units fail causing total system failure, consequently, the system may not be available for a long time and requirement of the system's operation does not allow long waiting time and hence some other substitute (may be cheaper or on rent) is called for continuation of operation with guarantee of failure free operation, until the repair of the failed system.

The present model may be understood by considering an electricity meter which may have connected to two phases, a main phase and an alternate phase. When main phase have any problem such as no power supply or any other failure then second phase that is alternate phase will start operating but following situations may also arrive when

- Both the phases have no power supply and both have cut of power supply.
- Electricity meter countered any problem such as fuse problem, short circuit etc.

If it is a fuse problem then it would be resolved so fast and system could be brought back to operative situation in no time but if any major failure occurs then an expert is called for repair and brings back the system to operative condition. This all exercise may be time consuming and may cause a heavy loss. To avoid it we may call for rented generator, battery or any other source for power supply with guarantee for failure free operation i.e. if the power source fails it has to be immediately replaced with the working one.

2. MODEL DESCRIPTION AND ASSUMPTIONS:

The system comprises two units, one unit is operative and other is kept as cold standby. The operation of any one unit is sufficient to do the job. Each unit of the system has two modes only normal operable mode and failure mode. Also, two types of repair facilities are considered. The following conditions are assumed.

1) Initially, one unit is operative and other is kept as cold standby.
2) On the failure of operative unit, standby unit becomes operative and failed unit will be under repair.
3) System/units are always repairable and repaired system/unit is as good as new.
4) The failure and repair time distributions are assumed to have exponential distributions.
5) When both the units fail causing total system failure, the following possibilities are considered:

i) If repair of a unit can be completed in small time then repair will be continued and the system is brought back to the operative condition.

ii) When both the units are failed and repair is taking more time than, some other substitute system (may be cheaper or on rent) is called for continuation of operation with guarantee of failure free operation to resume the desired operation. There may be a short period of downtime but the impact is much less than it would be otherwise. The substitute system is returned back only when the original system starts working as good as new after repair.

6) There are two types of repair facilities, one facility repairs a single unit if repairable in very short time and other facility repairs both the units, simultaneously.

3. NOTATIONS

O	operative unit
S	cold standby
F_r	unit is under repair
F_{wr}	failed unit is waiting for repair
F_{cr}	repair is continuing from previous state
C	connected substitute system
α_1	Failure rate of initially operative unit
α_2	Failure rate of a standby unit
Υ	continuous repair rate of a system
β_1	repair rate of the system i.e. both the units
β_2	repair rate of a unit
λ	rate of connecting substitute system

States of the System

The system may be in one of the following states:

S_0: (O, S) S_1: (F_r, O)
S_2: (F_r, F_{wr}) S_3: (F_{cr}, F_{wr})
S_4: (C)

With possible transitions the transition diagram is shown in Figure 1.

4. RELIABILITY AND MTSF:

The mean time to system failure (MTSF) for the proposed system is evaluated using the linear first order differential equations. Let $P_i(t)$ be the probability that the system at time t, $(t \geq 0)$ is in state S_i. Let P (t) denote the probability row vector at time t, the initial conditions for this problem are:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0)] = [1, 0, 0, 0, 0]$$

By employing the method of linear first order differential equations, we obtain the following differential equations:

$$\frac{dP_0}{dt} = -\alpha_1 P_0 + \beta_2 P_1 + \beta_1 P_4$$

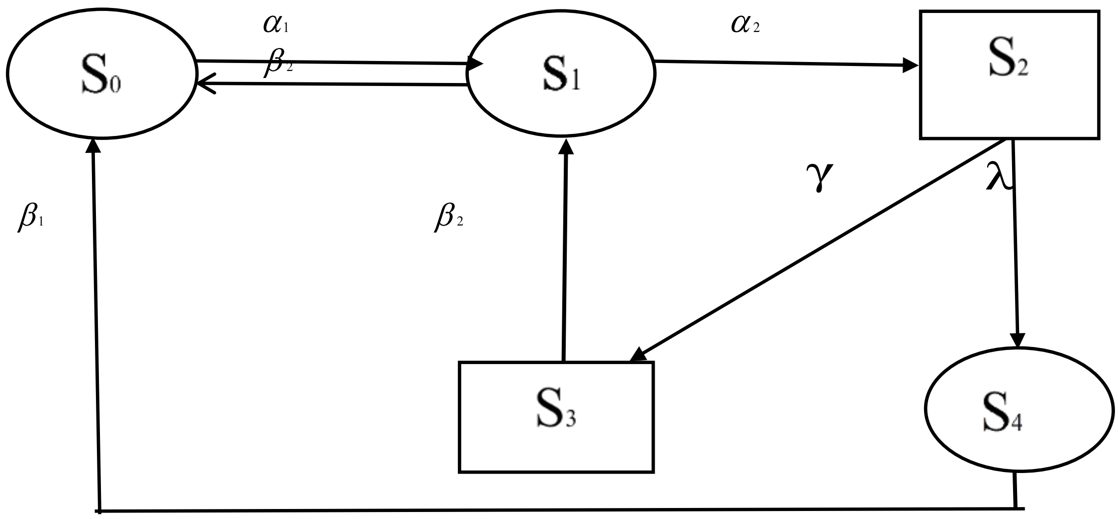

figure. 1

$$\frac{dP_1}{dt} = -(\alpha_2 + \beta_2) P_1 + \beta_2 P_3 + \alpha_1 P_0$$

$$\frac{dP_2}{dt} = -(\gamma + \lambda) P_2 + \alpha_2 P_1$$

$$\frac{dP_3}{dt} = -\beta_2 P_3 + \gamma P_2$$

$$\frac{dP_4}{dt} = -\beta_1 P_4 + \lambda P_2$$

... (1)

This can be written in the matrix form as:

$$P^* = Q P \qquad \dots (2)$$

where

$$Q = \begin{bmatrix} -\alpha_1 & \beta_2 & 0 & 0 & \beta_1 \\ \alpha_1 & -(\beta_2+\alpha_2) & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & \gamma & -\beta_2 & 0 \\ 0 & 0 & \lambda & 0 & -\beta_1 \end{bmatrix}$$

To calculate the MTSF, we take the transpose of the matrix Q and delete the rows and columns for the absorbing states. The new matrix is denoted by A. The expected time to reach an absorbing state is calculated from

$$E[T_{P(0) \rightarrow P(absorbing)}] = P(0)(-A^{-1}) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \dots (3)$$

where

$$A = \begin{bmatrix} -\alpha_1 & \alpha_1 & 0 \\ \beta_2 & -(\beta_2+\alpha_2) & 0 \\ \beta_1 & 0 & -\beta_1 \end{bmatrix}$$

We obtain the following expression for MTSF on solving equation (3).

$$MTSF = \frac{[\beta_2 + \alpha_1 + \alpha_2]}{\alpha_1 \alpha_2}.$$

5. AVAILABILITY ANALYSIS OF THE SYSTEM:

The initial conditions for this problem are same as for the reliability case:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0)] = [1, 0, 0, 0, 0]$$

The differential equations can be expressed as P *= Q P

$$\begin{bmatrix} P_0^* \\ P_1^* \\ P_2^* \\ P_3^* \\ P_4^* \end{bmatrix} = \begin{bmatrix} -\alpha_1 & \beta_2 & 0 & 0 & \beta_1 \\ \alpha_1 & -(\beta_2+\alpha_2) & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & \gamma & -\beta_2 & 0 \\ 0 & 0 & \lambda & 0 & -\beta_1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

In the steady-state situation, the derivatives of the state probabilities become zero. That is

$$QP(\infty) = 0 \qquad \dots (4)$$

That allows us to calculate the steady-state availability of the system as

$$A(\infty) = P_0(\infty) + P_1(\infty) + P_4(\infty)$$

Or ... (5)

$$A(\infty) = 1 - (P_2(\infty) + P_3(\infty))$$

Then the matrix equation becomes

$$\begin{bmatrix} -\alpha_1 & \beta_2 & 0 & 0 & \beta_1 \\ \alpha_1 & -(\beta_2+\alpha_2) & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & \gamma & -\beta_2 & 0 \\ 0 & 0 & \lambda & 0 & -\beta_1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad \dots (6)$$

Thus to obtain $P_0(\infty)$, $P_1(\infty)$, $P_2(\infty)$, $P_3(\infty)$, $P_4(\infty)$ we solve (6) under the normalizing condition

$$\sum_{i=0}^{4} P_i(\infty) = 1 \qquad \dots (7)$$

On substituting (7) in any one of the redundant rows in (6) which yield the following

$$\begin{bmatrix} -\alpha_1 & \beta_2 & 0 & 0 & \beta_1 \\ \alpha_1 & -(\beta_2+\alpha_2) & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & \gamma & -\beta_2 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \qquad \dots (8)$$

The solution of (8) provides the steady-state probabilities of the availability for the proposed system, i.e. $A(\infty)$ is given by

$$A(\infty) = 1 - (P_2(\infty) + P_3(\infty))$$

$$A(\infty) = 1 - \left(\frac{N_1}{D}\right) \qquad \dots (9)$$

$$N_1 = \alpha_1 \alpha_2 \beta_1 (\beta_2 + \gamma)$$

$$D = \beta_2 \{ \beta_1 [\alpha_1 (\lambda + \gamma + \alpha_2) + \lambda \alpha_2] + \alpha_1 \alpha_2 \lambda \} + \beta_1 \beta_2^2 (\lambda + \gamma)$$

6) BUSY PERIOD ANALYSIS FOR REPAIR TIME

The initial conditions for this problem are same as for the reliability case:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0)] = [1, 0, 0, 0, 0]$$

The differential equations can be expressed as P *= Q P

$$\begin{bmatrix} P_0^* \\ P_1^* \\ P_2^* \\ P_3^* \\ P_4^* \end{bmatrix} = \begin{bmatrix} -\alpha_1 & \beta_2 & 0 & 0 & \beta_1 \\ \alpha_1 & -(\beta_2+\alpha_2) & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & \gamma & -\beta_2 & 0 \\ 0 & 0 & \lambda & 0 & -\beta_1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} \quad ...(10)$$

Let B (∞) be the probability that the repair man is busy in repairing the failed unit then the steady-state busy period is given by

$B(\infty) = P_1(\infty) + P_2(\infty) + P_3(\infty)$

or

$B(\infty) = 1 - (P_0(\infty) + P_4(\infty))$ … (11)

In steady state, the derivatives of state probabilities become zero, thus (10) becomes

$QP(\infty) = 0$

Then the matrix equation becomes

$$\begin{bmatrix} -\alpha_1 & \beta_2 & 0 & 0 & \beta_1 \\ \alpha_1 & -(\beta_2+\alpha_2) & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & \gamma & -\beta_2 & 0 \\ 0 & 0 & \lambda & 0 & -\beta_1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad ...(12)$$

Thus to obtain $P_0(\infty)$, $P_1(\infty)$, $P_2(\infty)$, $P_3(\infty)$, $P_4(\infty)$ we solve (12) under the normalizing condition

$$\sum_{i=0}^{4} P_i(\infty) = 1 \quad … (13)$$

On substituting (13) in any one of the redundant rows in (12) which yield the following

$$\begin{bmatrix} -\alpha_1 & \beta_2 & 0 & 0 & \beta_1 \\ \alpha_1 & -(\beta_2+\alpha_2) & 0 & \beta_2 & 0 \\ 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & \gamma & -\beta_2 & 0 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad ...(14)$$

We solve the system of linear equations in (14) to obtain the state probabilities $P_0(\infty)$, $P_4(\infty)$. Thus

$B(\infty) = 1 - (P_0(\infty) + P_4(\infty))$

$B(\infty) = 1 - \left(\frac{N_2}{D}\right)$

where

$N_2 = \beta_1\beta_2^2(\gamma+\lambda) + \beta_1\beta_2\alpha_2\lambda + \alpha_1\beta_2\alpha_2\lambda$

$D = \beta_2\{\beta_1[\alpha_1(\lambda+\gamma+\alpha_2) + \lambda\alpha_2] + \alpha_1\alpha_2\lambda\} + \beta_1\beta_2^2(\lambda+\gamma)$

7) COMPARISON WITH PARTICULAR CASE

When substitute system is not used then by availability expression obtained in section (5), we have

$$A(\infty) = 1 - \frac{\alpha_1}{\beta_2 + 2\alpha_1} \quad .$$

For the model comparison, the following set of parameters values are fixed for consistency

$0.01 \le \alpha_1 \le 0.07$, $\alpha_2 = 0.01$, $\beta_1 = 0.07$, $\beta_2 = 0.09$, Y= 0.01, $\lambda = 0.05$

The calculated values for availability with substitute system and without substitute system are shown in following table and plotted in Figure 2.

α_1	Availability with substitute system	Availability without substitute system
0.01	0.98334523	0.909090909
0.02	0.970142888	0.846153846
0.03	0.95942029	0.8
0.04	0.950538774	0.764705882
0.05	0.943061656	0.736842105
0.06	0.936680235	0.714285714
0.07	0.931170108	0.695652174

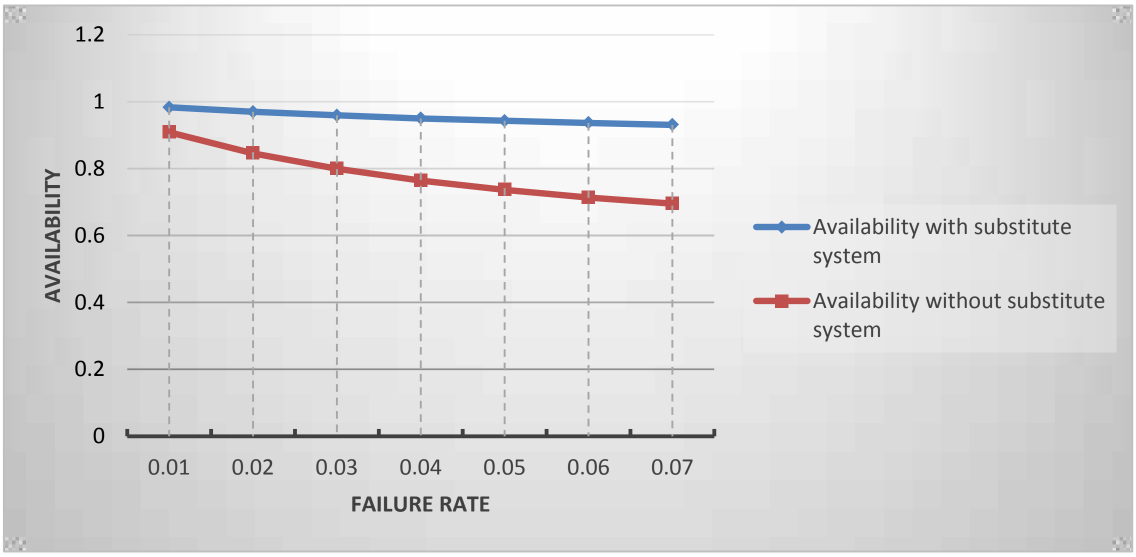

Figure .2

It is apparent from the above table and Figure 2 that availability is improved by incorporating the facility of substitute system. The proposed model not only improves the availability but also provides consistent availability.

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