AVAILABILITY IMPROVEMENT OF A SYSTEM WITH THREE MODES AND TWO TYPES OF REPAIR FACILITIES

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Abstract: This study deals with the reliability, availability, and busy period characteristics of two-unit cold standby system with three modes and two repair facilities. On failure of the system, if system can be repaired in short time then repair will be continued, otherwise in order to improve availability another substitute system taken from outside is used. The substitute system is guaranteed for failure free operation and an expert repairman is called for fast repair of the system. Assuming failure and repair times as exponentially distributed, expressions for the mean time to system failure (MTSF), the steady state availability and busy period for system are derived using linear first order differential equations. A particular case for the proposed system is discussed in which substitute system was not considered. Also, comparison is performed graphically to observe the effect of the proposed system on Availability.

Keywords: Availability, Linear first order differential equation, Mean Time to System Failure, Partial failure, Reliability, Steady State Availability.

1. INTRODUCTION:

Concept of two-unit cold standby system models assuming the three modes of each unit-Normal (N), Partial failure (P) and Total failure (F) have been studied by several authors including Mohammad El-Moniem Soleha who studied reliability and availability characteristics of a two-dissimilar-unit cold standby system with three modes by using linear first order differential equations. Goel L.R. and Gupta P. (1984) analyzed stochastic behavior of two unit hot standby system with three modes. Gupta P. and Kaur G. (2016) studied reliability and availability evaluation of a system switched to another similar, substitute or duplicate system on total failure. M.Y. Heggag (2009) analyzed cost of two-dissimilar-unit cold standby system with three modes and preventive maintenance by using linear first order differential equations. Goel L.R. and Gupta P. analyzed two-unit parallel system with partial and catastrophic failure and preventive maintenance. But very less attention is paid to improve availability. The present study is an effort to improve availability. Here, the system consists of a single unit with a cold standby unit with three modes and different failure rates. In present paper it is supposed that the system may also work in partially operative mode and standby unit becomes operative only when the unit fails totally. Repairman is available instantly on total failure of a unit. There may be a situation when both the units fail causing total system failure, consequently, the system may not be available for a long time and requirement of the system's operation does not allow long waiting time and hence some other substitute system (may be cheaper or on rent) is called for continuation of operation with guarantee of failure free operation, until the repair of the failed system. An expert repairman is called on total failure of system for fast repair, when substitute system is operational.

2. MODEL DESCRIPTION AND ASSUMPTIONS:

The system comprises two units. The operation of any one unit either in normal or partially operative mode is sufficient to do the job. Each unit of the system has three modes normal operable mode (N-mode), partial operable mode (P-mode), and complete failure mode (F-mode). Two types of repair facilities are considered. The following conditions are assumed.

1) Initially, one unit is operative and other is kept as cold standby.
2) Each unit first enters into P-mode and then into F-mode i.e. a unit can't enter into F-mode without passing through P-mode.

3) When a unit fails partially, repairman is intimated and be available with the system when a unit fails totally instantaneously.
4) System/units are always repairable and repaired system/unit is as good as new.
5) The failure and repair time distributions are assumed to have exponential distributions.
6) When both the units fail causing total system failure, the following possibilities are considered:

i) If undergoing repair of a unit can be completed in small (some specified) time then repair will be continued and the system is brought back to the operative condition.
ii) When repair is taking more than specified time, some other substitute system (may be cheaper or on rent) is called for continuation of operation with guarantee of failure free operation till repair of the system. The availability of system will be continued on employing substitute system. The substitute system is returned back only when the original system starts working as good as new after repair. In this case an expert repairman is called for early repair of the system.

3. NOTATIONS:

O	operative unit
S	cold standby
F_r	unit is under repair
F_{wr}	failed unit is waiting for repair
F_{cr}	repair is continuing from previous state
C	connected substitute system
α_1	failure rate of a unit from operational mode to partial mode
α_2	failure rate of a unit from partial mode to failure mode
ϒ	continuous repair rate of a system
β_1	repair rate of a system by expert repairman
β_2	repair rate of a unit by ordinary repairman
λ	rate of change of system

States of the System

The system may be in one of the following states:

S_0 (O, S) S_1 (P_o, S)
S_2 (F_{ur}, O) S_3 (F_{ur}, P_o)
S_4 (F_{ur}, F_{wr}) S_5 (F_{cr}, F_{wr})
S_6 (C)

The transition diagram is shown in Figure 1.

4. RELIABILITY AND MTSF

The mean time to system failure (MTSF) for the proposed system will be evaluated using the linear first order differential equations. Let P_i (t) is the probability that the system at time t, ($t \geq 0$) is in state S_i. Let P (t) denote the probability row vector at time t, the initial conditions for this problem are:

$$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)] = [1, 0, 0, 0, 0, 0, 0] \quad \ldots (4.1)$$

By employing the method of linear first order differential equations, we obtain the following differential equations:

$$\frac{dP_0}{dt} = -\alpha_1 P_0 + \beta_2 P_2 + \beta_1 P_6$$

$$\frac{dP_1}{dt} = -\alpha_2 P_1 + \alpha_1 P_0$$

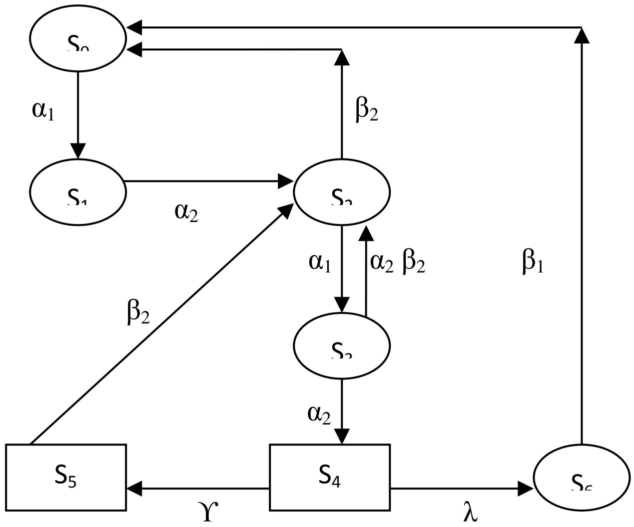

Figure 1

$$\frac{dP_2}{dt} = -(\alpha_1 + \beta_2)P_2 + \alpha_2 P_1 + \beta_2 P_5 + \beta_2\alpha_2 P_3$$

$$\frac{dP_3}{dt} = -(\alpha_2 + \beta_2\alpha_2)P_3 + \alpha_1 P_2$$

$$\frac{dP_4}{dt} = -(\gamma + \lambda)\, P_4 + \alpha_2 P_3$$

$$\frac{dP_5}{dt} = -\beta_2 P_5 + \gamma P_4$$

$$\frac{dP_6}{dt} = -\beta_1 P_6 + \lambda P_4$$

This can be written in the matrix form as:

$P^* = Q\,P$,

where

$$Q = \begin{bmatrix} -\alpha_1 & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 \\ \alpha_1 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & -(\alpha_1+\beta_2) & \beta_2\alpha_2 & 0 & \beta_2 & 0 \\ 0 & 0 & \alpha_1 & -(\alpha_2+\beta_2\alpha_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & -\beta_2 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & -\beta_1 \end{bmatrix}$$

To calculate the MTSF, we take the transpose of the matrix Q and delete the rows and columns for the absorbing state. The new matrix is called (A). The expected time to reach an absorbing state is calculated from

$$E[T_{P(0)\,\rightarrow\,P(\text{absorbing})}] = P(0)(-A^{-1})\begin{bmatrix}1\\1\\1\\1\\1\end{bmatrix}, \qquad \ldots (4.2)$$

where

$$A = \begin{bmatrix} -\alpha_1 & \alpha_1 & 0 & 0 & 0 \\ 0 & -\alpha_2 & \alpha_2 & 0 & 0 \\ \beta_2 & 0 & -(\alpha_1+\beta_2) & \alpha_1 & 0 \\ 0 & 0 & \beta_2\alpha_2 & -(\alpha_2+\beta_2\alpha_2) & 0 \\ \beta_1 & 0 & 0 & 0 & -\beta 1 \end{bmatrix}.$$

We obtain the following expression for MTSF on solving equation (4.2).

$$MTSF = \frac{-2\alpha_1^2 - 2\alpha_1\alpha_2 - \alpha_1\beta_2^2 - \alpha_2\beta_2^2 - \alpha_1\beta_2 - \alpha_1\alpha_2\beta_2 - \alpha_2\beta_2}{\alpha_1^2\alpha_2}$$

5. AVAILABILITY ANALYSIS OF THE SYSTEM:

The initial condition for this problem is same as for the reliability case i.e.

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0)] = [1, 0, 0, 0, 0, 0, 0]$

The differential equations can be expressed as

$P^* = Q P$

$$\begin{bmatrix} P^*_0 \\ P^*_1 \\ P^*_2 \\ P^*_3 \\ P^*_4 \\ P^*_5 \\ P^*_6 \end{bmatrix} = \begin{bmatrix} -\alpha_1 & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 \\ \alpha_1 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & -(\alpha_1+\beta_2) & \beta_2\alpha_2 & 0 & \beta_2 & 0 \\ 0 & 0 & \alpha_1 & -(\alpha_2+\beta_2\alpha_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & -\beta_2 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & -\beta_1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix}$$

In the steady-state situation, the derivatives of the state probabilities become zero. That is

$QP(\infty) = 0$

That allows us to calculate the steady-state availability of the system as

$A(\infty) = P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_6(\infty)$

Or

$A(\infty) = 1 - [(P_4(\infty) + P_5(\infty)]$ … (5.1)

Then the above matrix become

$$\begin{bmatrix} -\alpha_1 & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 \\ \alpha_1 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & -(\alpha_1+\beta_2) & \beta_2\alpha_2 & 0 & \beta_2 & 0 \\ 0 & 0 & \alpha_1 & -(\alpha_2+\beta_2\alpha_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & -\beta_2 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & -\beta_1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \ldots (5.2)$$

On substituting the normalizing condition $\sum_{i=0}^{6} P_i(\infty) = 1$, in any one of the redundant rows in (5.2) the following matrix is obtained

$$\begin{bmatrix} -\alpha_1 & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 \\ \alpha_1 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & -(\alpha_1+\beta_2) & \beta_2\alpha_2 & 0 & \beta_2 & 0 \\ 0 & 0 & \alpha_1 & -(\alpha_2+\beta_2\alpha_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & -\beta_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad \ldots (5.3)$$

The solution of (5.3) provides the steady-state probabilities of the availability for the proposed system, and then solution of (5.1) gives

$$A(\infty) = 1 - [P_4(\infty) + P_5(\infty)]$$

$$A(\infty) = 1 - \left(\frac{N_1}{D_1}\right)$$

Where

$$N_1 = \alpha_1^2\alpha_2\beta_1\beta_2 + \gamma\,\alpha_1^2\alpha_2\beta_1$$

$$\begin{aligned} D_1 = {} & \beta_2\beta_1\alpha_1^2(2\lambda+\alpha_2+\gamma) + \beta_2\beta_1\alpha_1\alpha_2(2\lambda+\gamma) + \beta_2\alpha_1^2\alpha_2\lambda + \beta_1\beta_2^2\alpha_1\alpha_2(\lambda+\gamma) \\ & + \beta_1\beta_2^2\alpha_1\lambda + \beta_1\beta_2^2\alpha_1\gamma + \beta_1\beta_2^2\alpha_2(\lambda+\gamma) + \beta_1\beta_2^3\alpha_2(\lambda+\gamma) + \beta_1\beta_2^3\alpha_1(\lambda+\gamma) \\ & + \gamma\,\alpha_1^2\alpha_2\beta_1 \end{aligned}$$

6. BUSY PERIOD ANALYSIS:

The initial condition for this problem is same as for the reliability case i.e. (4.1)

The differential equations can be expressed as

$$P^* = Q\,P$$

$$\begin{bmatrix} P^*_0 \\ P^*_1 \\ P^*_2 \\ P^*_3 \\ P^*_4 \\ P^*_5 \\ P^*_6 \end{bmatrix} = \begin{bmatrix} -\alpha_1 & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 \\ \alpha_1 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & -(\alpha_1+\beta_2) & \beta_2\alpha_2 & 0 & \beta_2 & 0 \\ 0 & 0 & \alpha_1 & -(\alpha_2+\beta_2\alpha_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & -\beta_2 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & -\beta_1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix} \quad .. (6.1)$$

Let B (∞) be the probability that the repair man is busy in repairing the failed unit then the steady- state busy period is given by

$$B(\infty) = P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty)$$

Or

$$B(\infty) = 1 - [P_0(\infty) + P_6(\infty)]$$

In steady state, the derivatives of state probabilities become zero, i.e.

$$QP(\infty) = 0$$

Then matrix (6.1) becomes

$$\begin{bmatrix} -\alpha_1 & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 \\ \alpha_1 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & -(\alpha_1+\beta_2) & \beta_2\alpha_2 & 0 & \beta_2 & 0 \\ 0 & 0 & \alpha_1 & -(\alpha_2+\beta_2\alpha_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & -\beta_2 & 0 \\ 0 & 0 & 0 & 0 & \lambda & 0 & -\beta_1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \ldots (6.2)$$

On substituting the normalizing condition $\sum_{i=0}^{6} P_i(\infty)=1$, in any one of the redundant rows in (6.2) the following matrix is obtained

$$\begin{bmatrix} -\alpha_1 & 0 & \beta_2 & 0 & 0 & 0 & \beta_1 \\ \alpha_1 & -\alpha_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha_2 & -(\alpha_1+\beta_2) & \beta_2\alpha_2 & 0 & \beta_2 & 0 \\ 0 & 0 & \alpha_1 & -(\alpha_2+\beta_2\alpha_2) & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha_2 & -(\gamma+\lambda) & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma & -\beta_2 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

We solve the system of linear equations in matrix above to obtain the state probabilities $P_0(\infty)$, $P_6(\infty)$.

$B(\infty) = 1 - (P_0(\infty) + P_6(\infty))$

$B(\infty) = 1 - \left(\frac{N_2}{D_1}\right)$

Where

$$N_2 = \alpha_2\beta_1\beta_2^3(\lambda+\gamma) + \alpha_2\beta_1\beta_2^2(\lambda+\gamma) + \alpha_1\alpha_2\beta_1\beta_2\lambda + \alpha_1^2\alpha_2\beta_2\lambda$$

$$\begin{aligned} D_1 = {} & \beta_2\beta_1\alpha_1^2(2\lambda+\alpha_2+\gamma) + \beta_2\beta_1\alpha_1\alpha_2(2\lambda+\gamma) + \beta_2\alpha_1^2\alpha_2\lambda + \beta_1\beta_2^2\alpha_1\alpha_2(\lambda+\gamma) \\ & + \beta_1\beta_2^2\alpha_1\lambda + \beta_1\beta_2^2\alpha_1\gamma + \beta_1\beta_2^2\alpha_2(\lambda+\gamma) + \beta_1\beta_2^3\alpha_2(\lambda+\gamma) + \beta_1\beta_2^3\alpha_1(\lambda+\gamma) \\ & + \gamma\alpha_1^2\alpha_2\beta_1 \end{aligned}$$

7. PARTICULAR CASE:

When substitute system is not used then on calculating availability as in section 5 we get

$$A(\infty) = \frac{N}{D}$$

Where,

$$N = \alpha_2\beta_2^3 + \alpha_2\beta_2^2 + \alpha_1\beta_2^3 + \alpha_1\beta_2^2 + \alpha_2\alpha_1\beta_2^2 + \alpha_2\alpha_1\beta_2 + \alpha_1^2\beta_2$$

$$D = (\alpha_2+\alpha_1)\beta_2^3 + (\alpha_2\alpha_1+\alpha_2+\alpha_1)\beta_2^2 + (\alpha_2\alpha_1+\alpha_1^2)\beta_2 + \alpha_2\alpha_1^2$$

8. NUMERICAL ILLUSTRATION AND CONCLUSION:

In this section, we numerically obtained and compare the results for system availability. The objectives here are to analyze graphically the effects of substitute system on availability for the model comparison, the following set of parameters values are fixed for consistency.

$0.01 \le \alpha_1 \le 0.04$, $\alpha_2 = 0.02$, $\beta_1 = 0.09$, $\beta_2 = 0.07$, Y= 0.05, $\lambda = 0.01$

The calculated values for availability with substitute system and without substitute system are shown in following table and Figure 2 shows the availability results for the system with and without substitute system being used with respect to α_1.

α_1	Availability with substitute system	Availability without substitute system
0.01	0.995086	0.988966
0.02	0.981205	0.970981
0.03	0.960588	0.953619
0.04	0.936026	0.934828

It is clear from the Figure 2 that system with substitute system has more availability with respect to α compared with the system without substitute system. These tend to suggest that system with substitute system is better than the other systems.

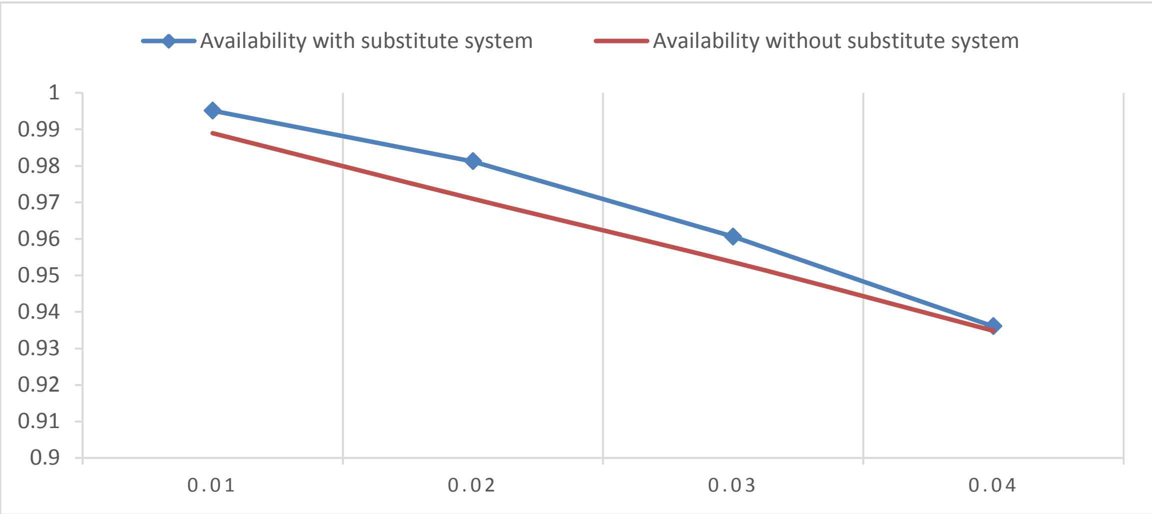

Figure 2

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