AVAILABILITY IMPROVEMENT IN SINGLE UNIT SYSTEM WITH TWO TYPES OF REPAIR FACILITIES

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Abstract: This study deals with the reliability, availability, and busy period characteristics of single unit system. On failure of the system, if system can be repaired in short time then repair will be continued, otherwise in order to improve availability another substitute system taken from outside is used, which is guaranteed for failure free operation and an expert repairman is called for fast repair of the unit. Assuming failure and repair times as exponentially distributed, expressions for the mean time to system failure (MTSF), the steady state availability and busy period for system are derived using linear first order differential equations. A particular case for the proposed system is discussed in which substitute system was not considered. Also comparison is performed graphically to observe the effect of the proposed system on Availability.
Keywords: Availability, Linear first order differential equation, Mean Time to System Failure, Reliability, Steady State Availability.

1. INTRODUCTION:

Competition exists in every field, to keep ahead a major challenge is availability improvement of a system, as less availability has negative impact. People often use "availability" and "reliability" interchangeably. In fact, however, the two terms are related but have distinct meanings. Reliability (as measure of the mean time between system failures, or MTBF) is one of two key components of availability. The other is the mean time required to repair a given system when it fails, or MTTR. The formula for availability is as follows:

Availability = MTBF / (MTBF + MTTR)

We can have example of a power supply system which is highly reliable, because it rarely experiences downtime, but not highly available because it has a high mean time to repair.

Many authors studied single unit systems, cold standby systems, warm standby systems etc. to improve reliability. But very less attention was paid to improve availability. The present study is an effort to improve availability, here the system consists of a single unit. When system fails, the system may not be available for a long time and requirement of the system's operation does not allow long waiting time and hence some other substitute (may be cheaper or on rent) is called for continuation of operation with guarantee of failure free operation, until the repair of the failed system.

The present model may be understood by considering an electricity meter with single phase. When power supply fails then following situations may arrive

- Phase may have a cut of power supply or any other sever problem.
- Electricity meter countered any minimal problem such as fuse problem.

If it is a fuse problem then it would be resolved so fast and system could be brought back to operative situation in no time but if any major failure occurs then an expert is called for repair and brings back the system to operative condition. This all exercise may be time consuming and may cause a heavy loss. To avoid it we may call for rented generator, battery or any other source for power supply with guarantee for failure free operation i.e. if the power source fails it has to be immediately replaced with the working one.

2. MODEL DESCRIPTION AND ASSUMPTIONS:

The system comprises a single unit. The system has two modes only normal operable mode and failure mode. The following conditions are assumed.

1) System/units are always repairable and repaired system/unit is as good as new.
2) The failure and repair time distributions are assumed to have exponential distributions.

3) When the units fail causing total system failure, the following possibilities are considered:

i) If repair of a unit can be completed in small time then repair will be continued and the system is brought back to the operative condition.

ii) When the unit is failed and repair is taking more time than, some other substitute system (may be cheaper or on rent) is called for continuation of operation with guarantee of failure free operation to resume the desired operation. There may be a short period of downtime but the impact is much less than it would be otherwise. The substitute system is returned back only when the original system starts working as good as new after repair.

2.1) NOTATIONS

O	operative unit
F	failed system
C	connected substitute system
α	failure rate of system
ϒ	repair rate by expert repairman
β	repair rate of the system
λ	rate of connecting substitute system

2.2) States of the System

The system may be in one of the following states:

S_0: (O) S_1: (F)

S_2: (C)

With possible transitions the transition diagram is shown in Figure 1.

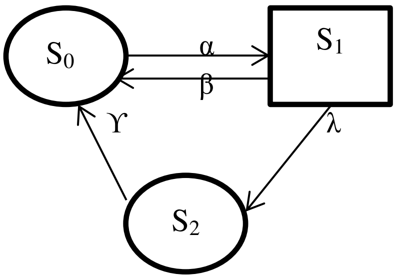

Figure 1

3. RELIABILITY AND MTSF:

The mean time to system failure (MTSF) for the proposed system is evaluated using the linear first order differential equations. Let $P_i(t)$ be the probability that the system at time t, $(t \geq 0)$ is in state S_i. Let P (t) denote the probability row vector at time t, the initial conditions for this problem are:

$$P(0) = [P_0(0), P_1(0), P_2(0)] = [1, 0, 0]$$

By employing the method of linear first order differential equations, we obtain the following differential equations:

$$\frac{dP_0}{dt} = -\alpha P_0 + \beta P_1 + \gamma P_2$$

$$\frac{dP_1}{dt} = \alpha P_0 - (\lambda + \beta) P_1$$

$$\frac{dP_2}{dt} = \lambda P_1 - \gamma P_2 \quad \text{... (1)}$$

This can be written in the matrix form as:

$$P^* = Q P \quad \text{... (2)}$$

where

$$Q = \begin{bmatrix} -\alpha & \beta & \gamma \\ \alpha & -(\beta+\lambda) & 0 \\ 0 & \lambda & \gamma \end{bmatrix}$$

To calculate the MTSF, we take the transpose of the matrix Q and delete the rows and columns for the absorbing states. The new matrix is denoted by A. The expected time to reach an absorbing state is calculated from

$$E[T_{P(0) \rightarrow P(absorbing)}] = P\,(0)\,(-A^{-1}) \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad \text{... (3)}$$

Where

$$A = \begin{bmatrix} -\alpha & 0 \\ \gamma & \lambda \end{bmatrix}$$

We obtain the following expression for MTSF on solving equation (3).

$$MTSF = \frac{1}{\alpha}.$$

4. AVAILABILITY ANALYSIS OF THE SYSTEM:

The initial conditions for this problem are same as for the reliability case:

$$P\,(0) = [P_0\,(0), P_1\,(0), P_2\,(0)] = [1, 0, 0]$$

The differential equations can be expressed as P *= Q P

$$\begin{bmatrix} P_0^* \\ P_1^* \\ P_2^* \end{bmatrix} = \begin{bmatrix} -\alpha & \beta & \gamma \\ \alpha & -(\beta+\lambda) & 0 \\ 0 & \lambda & \gamma \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix}$$

In the steady-state situation, the derivatives of the state probabilities become zero. That is

$$QP\,(\infty) = 0 \qquad \text{… (4)}$$

That allows us to calculate the steady-state availability of the system as

$$A\,(\infty) = 1 - P_1\,(\infty) \qquad \text{... (5)}$$

Then the matrix equation becomes

$$\begin{bmatrix} -\alpha & \beta & \gamma \\ \alpha & -(\beta+\lambda) & 0 \\ 0 & \lambda & \gamma \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad \text{...(6)}$$

Thus to obtain $P_0\,(\infty)$, $P_1\,(\infty)$, $P_2\,(\infty)$, $P_3\,(\infty)$, $P_4\,(\infty)$ we solve (6) under the normalizing condition

$$\sum_{i=0}^{4} P_i\,(\infty) = 1 \qquad \text{…. (7)}$$

On substituting (7) in any one of the redundant rows in (6) which yield the following

$$\begin{bmatrix} -\alpha & \beta & \gamma \\ \alpha & -(\beta+\lambda) & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \text{...(8)}$$

The solution of (8) provides the steady-state probabilities of the availability for the proposed system, i.e. A (∞) given by (5)

$$A(\infty) = \frac{\gamma(\alpha+\beta+\lambda)+\alpha\lambda - \alpha\gamma}{\gamma(\alpha+\beta+\lambda)+\alpha\lambda}$$

5. BUSY PERIOD ANALYSIS FOR FIRST TYPE AND SECOND TYPE OF REPAIR:

Let $B_1(\infty)$ and $B_2(\infty)$ be the busy period for first type and second type of repair and is given by equation (9), (10) respectively.

$$B_1(\infty) = P_1(\infty) \qquad \text{… (9)}$$

$$B_2(\infty) = P_2(\infty) \qquad \text{… (10)}$$

Initial conditions are same as availability then by solution of equation (8) in (9) and (10) we have

$$B_1(\infty) = \frac{\alpha\lambda}{\gamma(\alpha+\beta+\lambda)+\alpha\lambda}$$

$$B_2(\infty) = \frac{\alpha\gamma}{\gamma(\alpha+\beta+\lambda)+\alpha\lambda}$$

6. COMPARISON WITH PARTICULAR CASE:

When substitute system is not used then by availability expression obtained in section (5), we have

$$A(\infty) = \frac{\beta}{\beta+\alpha}$$

For the model comparison, the following set of parameters values are fixed for consistency

$0.01 \le \alpha \le 0.07$, $\beta = 0.07$, Y= 0.09, $\lambda = 0.01$

The calculated values for availability with substitute system and without substitute system are shown in following table and plotted in Figure 2.

α	Availability with substitute system	Availability without substitute system
0.01	0.890244	0.875
0.02	0.804348	0.777777778
0.03	0.735294	0.7
0.04	0.678571	0.636363636
0.05	0.631148	0.583333333
0.06	0.590909	0.538461538
0.07	0.556338	0.5

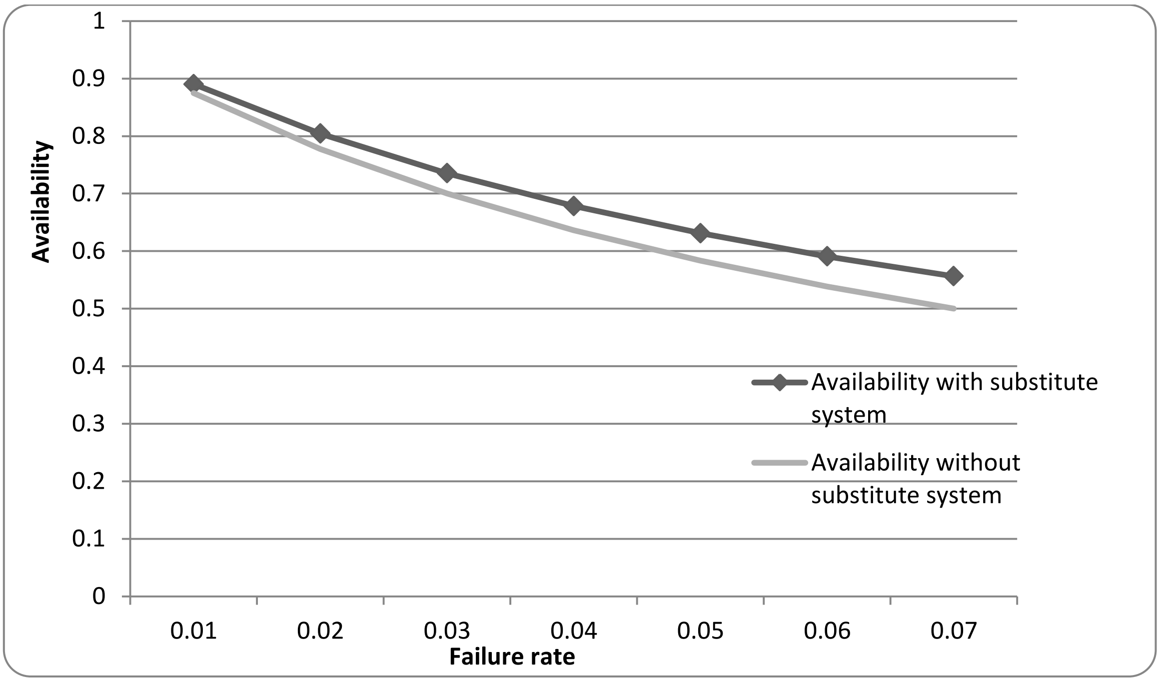

Figure 2

It is apparent from the above table and Figure 2 that availability is improved by incorporating the facility of substitute system.

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