RELIABILITY ANALYSIS OF A TWO- NON-IDENTICAL UNITS COLD STANDBY REPAIRABLE SYSTEM WITH SWITCHING OF UNITS BY USING LINEAR FIRST ORDER DIFFERENTIAL EQUATIONS

Pooja Vinodiya[1], **Gurvindar kaur**[2]
[1 & 2] School of Studies in Statistics,Vikram University, Ujjain (M.P.)
Email - poojavinodia10@gmail.com

Abstract: *This paper deals with a two non-identical unit system model in which one unit is operative and second is standby. These units interchange randomly. Failure and repair rates of both the units are different as they are non-identical. Whenever an operative units fails a switching device is used to disconnect the failed unit of the system. After disconnecting, the failed unit immediately goes for repair and the standby unit becomes operative. Linear differential technique is used for the analysis. Expressions for availability, MTSF and busy period are obtained.*

Key Words: *Reliability, Availability, Linear First Order Differential Equation, Mean Time to System Failure, Busy Period Analysis and Exponential Distribution.*

1. INTRODUCTION:

Several authors including Gupta et al. have studied the stochastic behaviour of two unit standby systems using regenerative point technique, linear differential equation etc. Gupta and Pandya (1998) has analysed a system model with inspection and adjustable rates. Recently Gupta and Kaur have evaluated reliability and availability using a substitute system. Linear first order differential equations are used for the analysis. Chaudhary and Tyagi considered a cold standby system in which the correlated failure and repair time distribution are consider. In practical situation the correlated failure and repair times are rarely observe. We in the present paper consider a two unit cold standby system in which the role of operation is changed randomly when both the unit are in normal operable conditions. The purpose of present study is to analyse the system when a switching device is used to disconnect the failed unit for repair.

2. MODEL DESCRIPTION:

This paper introduces statistical analysis of a two- dissimilar unit cold standby system, for which reliability and availability are calculated. The system has two non-identical units in cold standby configuration. Both the units have two modes Normal (N) and Failure (F). In the first state unit-1 is operative and unit-2 is kept as cold standby. Operative and standby units interchange randomly with constant rate. For this process no switching device is require units. Whenever an operative unit fails a switching device is used to disconnect the failed unit of the system and make a standby unit operative. A single repairman is always available with the system.

3. NOTATIONS AND STATES

Symbols for the states of the system

N_0 = N-mode and Operative
N_S = N-mode and Standby
F_r = F-mode and under repair
F_w = F-mode and waiting for repair
SD_r =Switching device under repair

Up State:

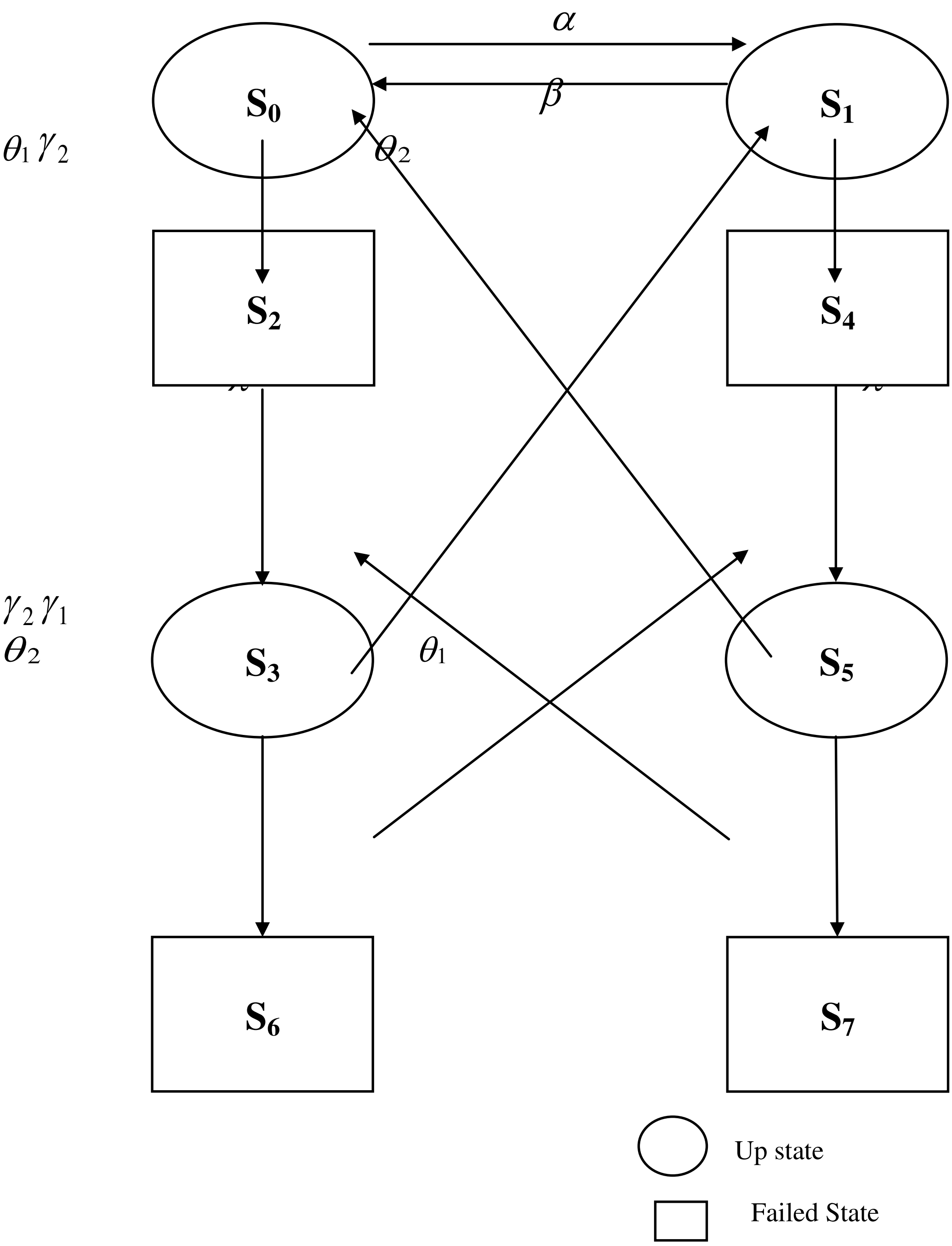
$S_0 = (N_0, N_S)$ $\quad$ $S_1 = (N_S, N_0)$
$S_3 = (F_r, N_0)$ $\quad$ $S_5 = (N_0, F_r)$

Failed State:

$S_2 = (F_w, N_S, SD_r)$ $\quad$ $S_4 = (N_S, F_w, SD_r)$
$S_6 = (F_r, F_w)$ $\quad$ $S_7 = (F_w, F_r)$

α, β = Constant rate of interchange of units.
θ_1 = failure rate of first unit
θ_2 = failure rate of second unit
γ_1 = Repair rate of first unit
γ_2 = Repair rate of second unit
λ = Repair rate of switching device

4. TRANSITION DIAGRAM:



5. RELIABILITY AND MEAN TIME TO SYSTEM FAILURE:

The mean time to system failure for the proposed system is obtain using linear first order differential equations. Let $p_i(t)$ be the probability that the system is in state at the time t. The initial conditions for this problem are:

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)]$

= [1, 0, 0, 0, 0, 0, 0, 0]

By using linear first order differential equations we obtain the following differential equations:

$$\frac{dp_0(t)}{dt} = -(\theta_1+\alpha)\,p_0(t)+\beta\,p_1(t)+\gamma_2\,p_5(t)$$
$$\frac{dp_1(t)}{dt} = -(\theta_2+\beta)\,p_1(t)+\alpha\,p_0(t)+\gamma_1\,p_3(t)$$
$$\frac{dp_2(t)}{dt} = -\lambda\,p_2(t)+\theta_1\,p_0(t)$$
$$\frac{dp_3(t)}{dt} = -(\theta_2+\gamma_1)\,p_3(t)+\lambda\,p_2(t)+\gamma_2\,p_7(t)$$
$$\frac{dp_4(t)}{dt} = -\lambda\,p_4(t)+\theta_2\,p_1(t)$$
$$\frac{dp_5(t)}{dt} = -(\theta_1+\gamma_2)\,p_5(t)+\lambda\,p_4(t)+\gamma_1\,p_6(t)$$
$$\frac{dp_6(t)}{dt} = -\gamma_1\,p_6(t)+\theta_2\,p_3(t)$$
$$\frac{dp_7(t)}{dt} = -\gamma_2\,p_7(t)+\theta_1\,p_5(t) \qquad 1.$$

Differential equation (1) can be written in matrix form as follow:

$$P^* = QP \qquad 2.$$

Where

$$P^* = \begin{bmatrix} P_0^* \\ P_1^* \\ P_2^* \\ P_3^* \\ P_4^* \\ P_5^* \\ P_6^* \\ P_7^* \end{bmatrix}$$

$$Q = \begin{bmatrix} -(\theta_1+\alpha) & \beta & 0 & 0 & 0 & \gamma_2 & 0 & 0 \\ \alpha & -(\theta_2+\beta) & 0 & \gamma_1 & 0 & 0 & 0 & 0 \\ \theta_1 & 0 & -\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & -(\theta_2+\gamma_1) & 0 & 0 & 0 & \gamma_2 \\ 0 & \theta_2 & 0 & 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & -(\theta_1+\gamma_2) & \gamma_1 & 0 \\ 0 & 0 & 0 & \theta_2 & 0 & 0 & -\gamma_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_1 & 0 & -\gamma_2 \end{bmatrix}$$

$$P=\begin{bmatrix} P_0(t)\\ P_1(t)\\ P_2(t)\\ P_3(t)\\ P_4(t)\\ P_5(t)\\ P_6(t)\\ P_7(t)\end{bmatrix}$$

To calculate the MTSF we take the we delete the row and columns of absorbing state of matrix Q and take the transpose to produce a new matrix A. The expected time to reach an absorbing state is calculated by

E [Tp(0) → p(observing)]

$$= p(0)\ \int_0^\infty e^{-At}\,dt \qquad 3.$$

$$= p(0)\,(-A^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix}$$

where

$$A=\begin{bmatrix} -(\theta_1+\alpha) & \alpha & 0 & 0\\ \beta & -(\theta_2+\beta) & 0 & 0\\ 0 & \gamma_1 & -(\theta_2+\gamma_1) & 0\\ \gamma_2 & 0 & 0 & -(\theta_1+\gamma_2)\end{bmatrix}$$

By solving the equation 3 we get the following expression for the MTSF

$$MTSF = \frac{\alpha+\theta_2+\beta}{\theta_1\ \theta_2+\alpha\,\theta_2+\beta\,\theta_1}$$

6. AVAILABILITY ANALYSIS OF THE SYSTEM:

The initial conditions for this problems are

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)]$

$= [1, 0, 0, 0, 0, 0, 0, 0]$

The differential equation can be express as

$P^* = QP$

$$\begin{bmatrix} P_0^*\\ P_1^*\\ P_2^*\\ P_3^*\\ P_4^*\\ P_5^*\\ P_6^*\\ P_7^*\end{bmatrix}=\begin{bmatrix} -(\theta_1+\alpha) & \beta & 0 & 0 & 0 & \gamma_2 & 0 & 0\\ \alpha & -(\theta_2+\beta) & 0 & \gamma_1 & 0 & 0 & 0 & 0\\ \theta_1 & 0 & -\lambda & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \lambda & -(\theta_2+\gamma_1) & 0 & 0 & 0 & \gamma_2\\ 0 & \theta_2 & 0 & 0 & -\lambda & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \lambda & -(\theta_1+\gamma_2) & \gamma_1 & 0\\ 0 & 0 & 0 & \theta_2 & 0 & 0 & -\gamma_1 & 0\\ 0 & 0 & 0 & 0 & 0 & \theta_1 & 0 & -\gamma_2\end{bmatrix}\begin{bmatrix} P_0\\ P_1\\ P_2\\ P_3\\ P_4\\ P_5\\ P_6\\ P_7\end{bmatrix} \qquad 4.$$

In the steady state situation, the derivatives of the state probabilities become zero. That is

$QP(\infty)=0$

That allows us to calculated the steady state availability of the system as

$A(\infty) = [P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty)]$ or
$A(\infty) = 1 - [P_2(\infty) + P_4(\infty) + P_6(\infty) + P_7(\infty)]$

Then the equation 4. Can be written as:

$$\begin{bmatrix} -(\theta_1+\alpha) & \beta & 0 & 0 & 0 & \gamma_2 & 0 & 0 \\ \alpha & -(\theta_2+\beta) & 0 & \gamma_1 & 0 & 0 & 0 & 0 \\ \theta_1 & 0 & -\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & -(\theta_2+\gamma_1) & 0 & 0 & 0 & \gamma_2 \\ 0 & \theta_2 & 0 & 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & -(\theta_1+\gamma_2) & \gamma_1 & 0 \\ 0 & 0 & 0 & \theta_2 & 0 & 0 & -\gamma_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_1 & 0 & -\gamma_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad 5.$$

Thus to obtain $P_0(\infty),\ P_1(\infty),\ P_2(\infty),\ P_3(\infty),\ P_4(\infty), P_5(\infty),\ P_6(\infty),\ P_7(\infty)$ we solve this equation under the normalizing condition.

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty) = 1 \quad 6.$$

On substitute (7) in the last row of (6) to give the following system of linear equations in matrix form

$$\begin{bmatrix} -(\theta_1+\alpha) & \beta & 0 & 0 & 0 & \gamma_2 & 0 & 0 \\ \alpha & -(\theta_2+\beta) & 0 & \gamma_1 & 0 & 0 & 0 & 0 \\ \theta_1 & 0 & -\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & -(\theta_2+\gamma_1) & 0 & 0 & 0 & \gamma_2 \\ 0 & \theta_2 & 0 & 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & -(\theta_1+\gamma_2) & \gamma_1 & 0 \\ 0 & 0 & 0 & \theta_2 & 0 & 0 & -\gamma_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \quad 7.$$

The solution of (7) provides the steady state probability of the availability for the system. $A(\infty)$ is given by

$$A(\infty) = 1 - [P_2(\infty) + P_4(\infty) + P_6(\infty) + P_7(\infty)]$$

$$A(\infty) = 1 - \frac{N_1}{D} \quad 8.$$

Where

$$N_1 = (\alpha + \theta_1) + (\theta_2 + \beta) + (\theta_2 + \gamma_1) + (\theta_1 + \gamma_2)$$

$$D = (\theta_2+\gamma_1)(\theta_1+\gamma_2)(\theta_2+\beta) + \gamma_1\ \gamma_2\ (\theta_2+\gamma_1) + \beta\gamma_1(\theta_2+\gamma_1)\ (\theta_1+\gamma_2)$$

7. BUSY PERIOD ANALYSIS:

The initial conditions for busy period are

$P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0), P_5(0), P_6(0), P_7(0)]$
$= [1, 0, 0, 0, 0, 0, 0, 0]$

The differential equation can be express as

$P^* = QP$

$$\begin{bmatrix} P_0^* \\ P_1^* \\ P_2^* \\ P_3^* \\ P_4^* \\ P_5^* \\ P_6^* \\ P_7^* \end{bmatrix} = \begin{bmatrix} -(\theta_1+\alpha) & \beta & 0 & 0 & 0 & \gamma_2 & 0 & 0 \\ \alpha & -(\theta_2+\beta) & 0 & \gamma_1 & 0 & 0 & 0 & 0 \\ \theta_1 & 0 & -\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & -(\theta_2+\gamma_1) & 0 & 0 & 0 & \gamma_2 \\ 0 & \theta_2 & 0 & 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & -(\theta_1+\gamma_2) & \gamma_1 & 0 \\ 0 & 0 & 0 & \theta_2 & 0 & 0 & -\gamma_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_1 & 0 & -\gamma_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix}$$
9.

Let $B(\infty)$ be the probability that the repair man busy in repairing the failed unit. Then the busy period is given by

$B(\infty) = 1-[P_0(\infty) + P_1(\infty)]$ 10.

In steady state, the derivatives of state probabilities become zero, then equation (9) can be written as follow:

$$\begin{bmatrix} -(\theta_1+\alpha) & \beta & 0 & 0 & 0 & \gamma_2 & 0 & 0 \\ \alpha & -(\theta_2+\beta) & 0 & \gamma_1 & 0 & 0 & 0 & 0 \\ \theta_1 & 0 & -\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & -(\theta_2+\gamma_1) & 0 & 0 & 0 & \gamma_2 \\ 0 & \theta_2 & 0 & 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & -(\theta_1+\gamma_2) & \gamma_1 & 0 \\ 0 & 0 & 0 & \theta_2 & 0 & 0 & -\gamma_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \theta_1 & 0 & -\gamma_2 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
11.

Thus to obtain $P_0(\infty)$, $P_1(\infty)$, $P_2(\infty)$, $P_3(\infty)$, $P_4(\infty)$, $P_5(\infty)$, $P_6(\infty)$, $P_7(\infty)$ we solve this equation under the normalizing condition.

$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) + P_5(\infty) + P_6(\infty) + P_7(\infty) = 1$ 12.

On substitute (12) in the last row of (11) to give the following system of linear equations in matrix form

$$\begin{bmatrix} -(\theta_1+\alpha) & \beta & 0 & 0 & 0 & \gamma_2 & 0 & 0 \\ \alpha & -(\theta_2+\beta) & 0 & \gamma_1 & 0 & 0 & 0 & 0 \\ \theta_1 & 0 & -\lambda & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \lambda & -(\theta_2+\gamma_1) & 0 & 0 & 0 & \gamma_2 \\ 0 & \theta_2 & 0 & 0 & -\lambda & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \lambda & -(\theta_1+\gamma_2) & \gamma_1 & 0 \\ 0 & 0 & 0 & \theta_2 & 0 & 0 & -\gamma_1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$
13.

The solution of (13) provides the state probability for busy period. Thus

$B(\infty) = 1-[P_0(\infty) + P_1(\infty)]$

$$B(\infty) = 1-\frac{N_2}{D}$$

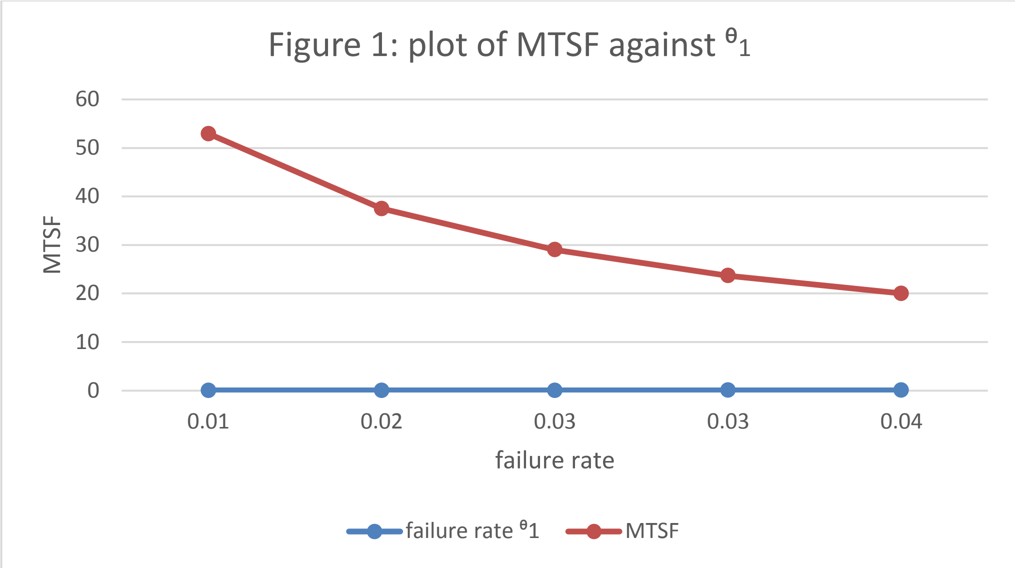
Where

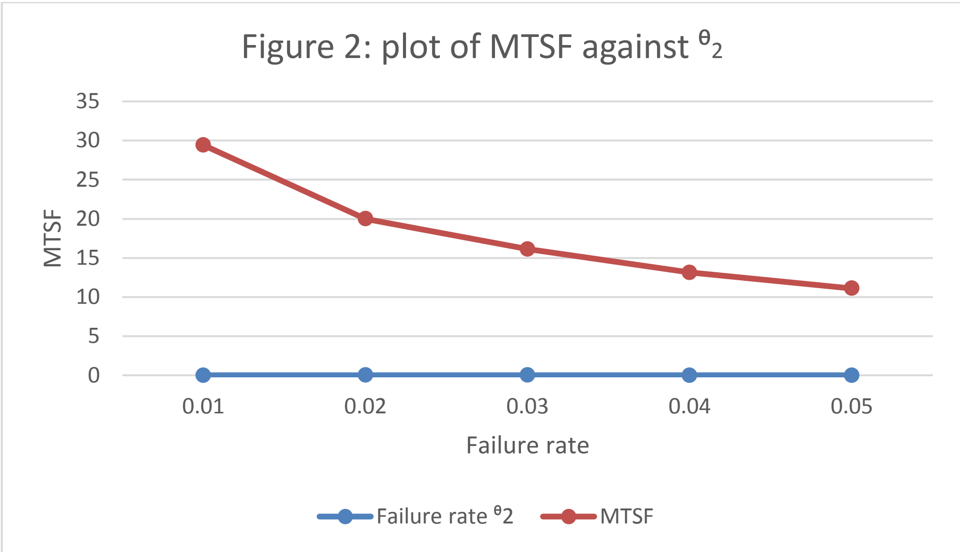
$N_2 = (\theta_2+\beta)+\gamma_2(\theta_2+\gamma_1)(\theta_2+\beta)+\gamma_1(\theta_1+\gamma_2)$

$D = (\theta_2+\gamma_1)(\theta_1+\gamma_2)(\theta_2+\beta)+\gamma_1\ \gamma_2\ (\theta_2+\gamma_1)+\beta\gamma_1(\theta_2+\gamma_1)\ (\theta_1+\gamma_2)$

8. GRAPHICAL STUDY OF THE SYSTEM:

For a more concrete study of the system behaviour, we plot curves for MTSF for the different values of θ_1 and θ_2 :





REFERENCES:

1. Gupta P. and Pandya R. (1998): "Reliability measures of two-unit redundant system with inspection policy and adjustable rates", Ultra Sciences. Vol. 10(2), 232-235.
2. Gupta P. and Kaur Gurvindar (2016): "Reliability and Availability evaluation of a system switched to another similar substitute or duplicate system on total failure", International Journal for Innovative Research in Multidisciplinary Field. Vol. 2(9), 57-62.
3. Goel,L,R. and Gupta Praveen (1983), "Stocastic behaviour of a two unit(dissimilar) hot standby system with three modes", Microelectron relib. Vol.23(6), 1035-1040.
4. Goel,L.R., Sharma, G.C. and Gupta Praveen (1985),"Cost benefit analysis of a system with intermittent repair and inspection under abnormal weather"Microelectron. Reliab.,vol. 2(5), 665-668.
5. Khaled M. El-Said and Mohamed salah El-Sherbeny (2005)"Evaluation of Reliability and Availability Characteristics of Two Different Systems by Using Linear First Order Differential Equations", Journal of Mathematics and Statistics. Vol.1(2),119-123.