Copula Approach: Correlation Between Bond Market and Stock Market, Between Developed and Emerging Economies

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Abstract: *Bond market and the stock market are interlinked with slightly positive correlation and the degree varies depending upon weather the economy is developed or emerging. Using daily returns of stock indices and bonds from a few emerging and developed countries, this paper intents to establish the correlation of bond market and stock market in few developed and developing markets. Since data is not normally distributed therefore copula is used to measure the correlations. US Bond market and the US stock market shows an upward trend stating that they move together. The scatter plot of China Bond market and China stock market shows almost stable trend representing that there is very less relation between the bond and stock market in China. Indian market analysis tells us that the correlation of the bond market and stock market is high.*

Key Words: *Correlations, Copula, Non-normality, Bond markets, Stock markets, JEL Classification: C20, C22*

1. INTRODUCTION:

In theoretical aspect, the bond market and the stock market are interlinked with slightly positive correlation and the degree varies depending upon weather the economy is developed or emerging. Correlations can be estimated by Kendall's tau, Spearman rho etc. As the contagion effect follows a non-linear pattern, copula strategy is more appropriate method to find the correlation between the two markets and understand the contagion effect among them. Using daily returns of stock indices and bonds from a few emerging and developed countries, this paper intents to establish the correlation of bond market and stock market in few developed and developing markets. Identifying and quantifying dependencies is the core of econometric modelling, and helps in the risk management. Therefore, to better understand the bond market and stock market dependencies correlation is calculated with the help of copula approach.

Rest of the paper is organized as follows, section 2 reviews the literature, section 3 describes the of data and methodology. Section 4 reports the results of the study and section 5 concludes the study.

2. LITERATURE REVIEW:

Hu (2006) studied the modelling and estimation of dependence across worldwide economic markets, with a focus on the structure of dependence. The methodology used is carried out to estimate the dependence throughout several globally present worldwide stock markets.

Rodriguez (2007) - This paper helps in creating a model of dependence with switching-parameter copulas to study contagion effect in the financial markets present globally.

Durante (2008) - They helps in defining the methods and investigating contagion effect among the two economic markets X and Y by using the information regarding their dependence, using the technique known as copula C(X,Y).

Pereira (2011) – They tests the hypothesis of contagion effect present between the indices of global financial markets. Results of this research conclude that both for the Gaussian copula and for the SJC (Symmetrized Joe Clayton) copula there is an evidence of contagion effect present among the financial markets.

Grover (2015)- He examined the equities in the Indian markets and concluded that assets returns in Indian markets do not follow an elliptical dependence structure using the technique of copula.

Jondeau & Rockinger (2002)- This research proposed a new methodology by joining copula functions with the GARCH technique which requires in first estimating the univariate distributions of the financial series and then joining this distribution with copula

3. DATA AND METHODOLOGY:

Index of major stocks and bonds market with daily records for past eleven years (2006- 2016) is taken in the study. Data is taken fr0m Bloomberg Terminal. Countries selected for analysis are: advanced economies: USA and Japan, emerging economies: India and China.

Traditional Correlation

Correlation is a bivariate evaluation that measures the strengths of association among two variables. In records, the fee of the correlation coefficient varies between +1 and -1. While the price of the correlation coefficient lies around ± 1, then it's far stated to be a perfect diploma of affiliation between the 2 variables, because the correlation coefficient value is going toward 0, the connection between the 2 variables might be weaker. Three varieties of correlations are:

Pearson correlation (r)

Pearson correlation is extensively used in statistics to degree the diploma of the connection among linear associated variables. Example: Within the stock marketplace, if we want to measure how stocks are associated with each other, Pearson correlation is used to degree the degree of relationship among the two stocks. The formula used to calculate the Pearson correlation:

$$r = \frac{N\sum xy - \sum x \sum y}{\sqrt{\{N\ \sum x^2 - (\sum x)^2\} * \{N\ \sum y^2 - (\sum y)^2\}}}$$

Where:

r = Pearson correlation coefficient

N = Number of values in data set

$\sum xy$ = Sum of the products of paired values

$\sum x$ = Sum of x values

$\sum y$ = Sum of y values

$\sum x^2$= Sum of squared x values

$\sum y^2$= Sum of squared y values

Pearson correlation has various assumptions such as both variables must be typically dispensed, linearity and homoscedasticity. Linearity assumes a straight line relationship between each of the within the analysis and homoscedasticity assumes that data is normally distributed about the regression line.

Kendall rank correlation (tau)

Kendall rank correlation is a non-parametric test that measures the strength of dependence between two variables. Taking two samples, where sample size is the total number of pairings with the two samples is $n(n-1)/2$.

The formula used:

$$\tau = \frac{N_c - N_d}{\frac{1}{2}n(n-1)}$$

Where

Nc = Number of concordant (When the order of samples is same)

Nd = Number of discordant (When the order of samples is different)

n = Sample size

The traditional method assumes an existence of a linear relationship or a similar relationship exists between independent and dependent variables. This method does not focus on the values of the distribution which are near the mean, and gives the concentrated results which needs to be more diversified. Hence, to find the contagion effect using extreme tail values for better prediction of the correlation between the two series, the new technique is developed known as copula.

COPULA TECHNIQUE

In the probability theory, a copula is a multivariate probability distribution which has uniform marginal distribution of variables. The copula technique is said to have started with Frechet, but in 1999 Sklar obtained the result by deriving the name copula and the concept with it. A copula is a multivariate distribution function from the unit d-cube

$$\boldsymbol{C(1, \dots, 1, u_i, 1, \dots, 1) = u_i \; \forall \; i \leq d \; and \; u_i \in [0, 1]}$$

This property means that if the realizations of the d-1 variables are known with marginal probability one, then the d outcomes of the joint probability is equal to the one with uncertain outcome (U_i).

$$\boldsymbol{C(u_1, \dots\dots, u_d) = 0 \; if \; u_i = 0 \; \forall \; i \; \leq d}$$

This property says that if the realization of one variable has the marginal probability zero than the joint probability of all outcomes is zero. This property is also known as the grounded property. This property ensures that the joint probability will be not negative. This is because the volume (C) of any d-dimensional interval is non-negative. The upper bound is called the Frechet -Hoeffding upper bound and the lower bound is called the Frechet- Hoeffding lower bound.

Benefits of Copula

Copula functions are parametrically targeted joint distributions generated from given marginal. This presents the key benefits by keeping the dependence structure and the univariate distributions of the variables apart. The copula approach provides a measure of isolating the marginal behavior and the dependence structure of variables from their joint distribution function. This separation explains the modelling flexibility furnished by copulas and the wide interest in copulas for modeling the dependence structure among the variables.

Types of Copula

The copula functions are classified in three main categories:

Fundamental Copula: These represent the perfect positive dependence, independence and perfect negative dependence.

Implicit Copula: They are extracted from well-known multivariate distributions and do not have closed form expressions.

Explicit Copula: Also known as **Archimedean copulas** are simple closed form expressions and follow general mathematical constructions to yield copulas.

Table 1: Types of Copula

Type of Archimedean Copula	Author	Year	Bivariate Copula $C_\theta(u,v)$	Parameter θ
Normal or Gaussian	Cherubini, Luciano & Vecchiato	2004	$\frac{1}{\theta}(\theta(u)+\theta(v))$	$\theta\epsilon\,[0,\infty)$
Frank	Frank	1979	$\frac{1}{\theta}\log[1+\frac{(\exp(-\theta u)-1)(\exp(-\theta v)-1)}{\exp(-\theta)-1}]$	$\theta\epsilon\, R\backslash\{0\}$
Gumbel	Emil Julius Gumbel	1960	$exp[-((-\log(u))^\theta+((-\log(v))^\theta)^{1/\theta}]$	$\theta\epsilon\,[1,\infty)$
Clayton	David George Clayton	1978	$[\max\{u^{-\theta}+v^{-\theta}-1;0\}]^{-1/\theta}$	$\theta\epsilon\,[-1,\infty)\backslash\{0\}$

4. RESULTS:

The study was focused on finding correlations among the major stock market and the bond market in the developing economies (India and China) and the developed economies (USA).

1) US Bond and US Stock index

The scatter plot of US Bond market and the US stock market shows an upward trend stating that they move together and both the stock and the bond market in the US have increased over the period.

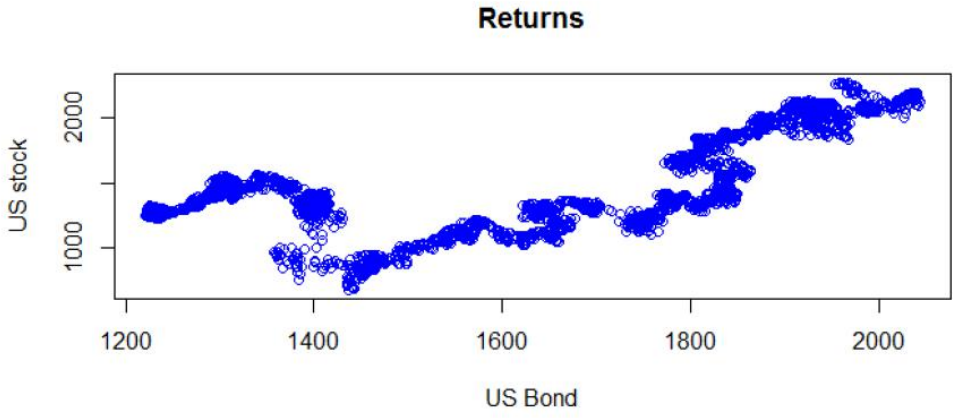

Figure 1: Plot of US stock and bond

Table 2: Traditional measure of correlations

Variable 1	Variable 2	Pearson	Kendall	Spearman
US Bond	US Stock	0.6906726	0.5232528	0.6865547

Table 3: Copula Measure

Copula Techniques	Estimate	Standard Error	MLE	AIC
Normal	0.647855	0.009293	745.3	-1488.63
Frank	5.1762	0.1391	776.9	-1551.805
Gumbel	1.93851	0.02943	993.6	-1985.183
Clayton	0.5002	0.02899	166.6	-331.132

The analysis shows that by the traditional method the correlation is similar in the values. However, the tails analysis i.e. the various copulas fit measures data differently. Looking at the results obtained from various copulas implemented we interpret that the Gumbel copula is a better fit model than the other copulas based on the MLE (Maximum Likelihood Estimation) and the AIC (Akaike Information Criterion).

2) China Bond and China Stock index

The scatter plot of China Bond market and China stock market shows almost stable trend representing that there is very less relation between the bond and stock market in China.

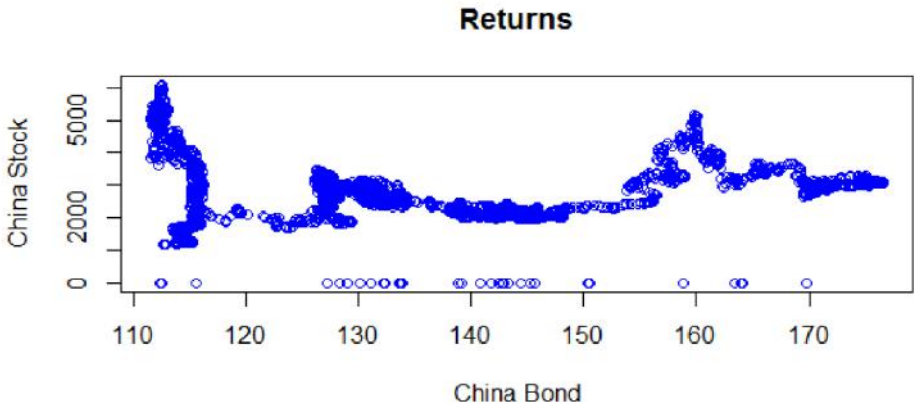

Figure 2: Plot of China stock and bond

Table 4. Traditional measure of correlation

Variable 1	Variable 2	Pearson	Kendall	Spearman
China Bond	China Stock	0.001014059	-0.005344277	0.02958155

Table 5: Copula measure

Copula Techniques	Estimate	Standard Error	MLE	AIC
Normal	-0.0244	0.01928	0.7990868	0.4018265
Frank	0.2199	0.127	1.494403	-0.988806
Gumbel	tau is out of the range [0, 1]			
Clayton	0.0195	0.02224	0.4020888	1.195822

The analysis by the traditional method the correlation is very low and the Kendall's tau comes out to be negative showing an opposite relation. But using, the copulas techniques of various types of copulas help us to find the best method which is Frank Copula based on the MLE (Maximum Likelihood Estimation).

3) India Bond and India Stock index

The scatter plot of Indian market data shows us an increasing trend, clearly stating that the Indian stock and Indian bond market moves together and increases with the time period.

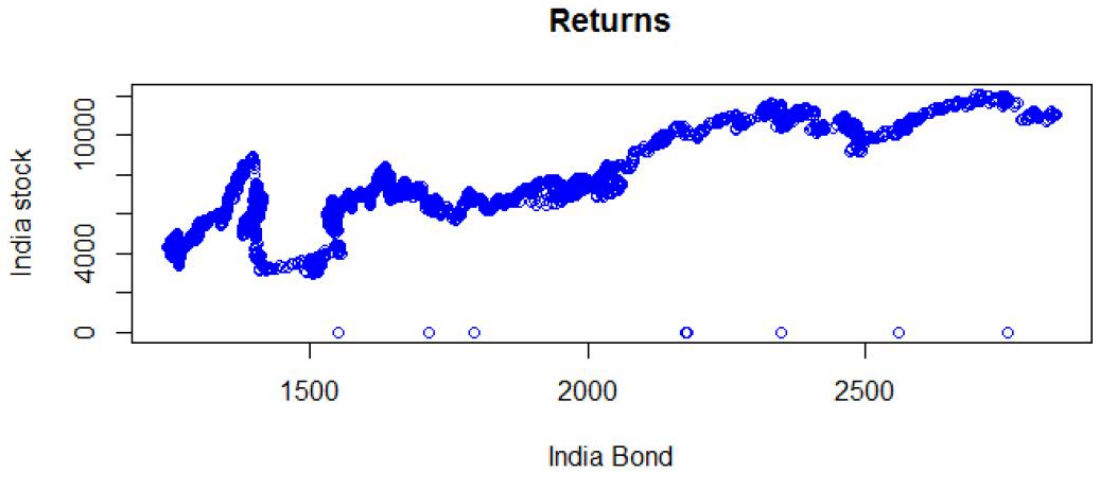

Figure 3: Plot of India stock and bond

Table 6

Variable 1	Variable 2	Pearson	Kendall	Spearman
India Bond	India Stock	0.8656148	0.6763665	0.8472291

Table 7

Copula Techniques	Estimate	Standard Error	MLE	AIC
Normal	0.812824	0.005021	1482	-2962.522
Frank	9.921	0.2018	1714.902	-3427.805
Gumbel	2.64266	0.04201	1628.084	-3254.168
Clayton	1.5806	0.0486	859.0126	-1716.025

Indian market analysis tells us that the correlation of the bond market and stock market is high. The fit of the copula techniques is determined by the help of MLE (Maximum Likelihood Estimate) stating that the Frank copula is a better measure than the traditional method.

5. CONCLUSION:

Copula approach involves specifying marginal distributions of each random variable along with a copula function that binds them together. The copula model has two key advantages: First, there were no assumptions made about the marginal distributions. There was no requirement that they should be normal distributions or that they should have the same distributions. The second advantage is the ability to separate the dependence structure from the marginal distributions. These advantages allow us to describe the same marginal distributions through difference Copulas functions and dependency structures. The paper uses four types of Archimedean copulas, Guassian Copula, Frank copula, Gumbel copula and Clayton copula to identify the correlation and tells us that among the countries analyzed

the We can also conclude based on the results that Frank and Gumbel copulas are the mostly fit copula model for the analyzed countries. US Bond market and the US stock market shows an upward trend stating that they move together. The scatter plot of China Bond market and China stock market shows almost stable trend representing that there is very less relation between the bond and stock market in China. Indian market analysis tells us that the correlation of the bond market and stock market is high.

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