Determination of the stress state and strength characteristics of the jaw's teeth during the packer's operation

[1] Mammadov V. T, [2] Shahnazarov M. A.
[1]Professor, [2]Head of Department
[1]Azerbaijan State Oil and Industry University, Baku, Azerbaijan,
[2]State Agency for Safe Working in Industry and Mountain-Mine Control Azerbaijan State Scientific-Research Institute for Labour Protection and Safety Measures, Baku, Azerbaijan
Email – [1]vasif.mamedov_adnsu@mail.ru, [2]mohsunsh@gmail.com

Abstract: *In the article the stress state and strength parameters of the jaw's teeth during the operation of packers areinvestigated.The calculation of the jaw's teeth of the packer unit is not justified according to the formula "Resistance of materials", since, in fact, only the jaw'steeth affect the inner surface of the column by the area of contact* [1]. *Therefore, the authors of the article propose a calculation method, in which transverse forces and bending moments are taken into account.In order to determine the potential energy, we used the deflection function described through the Laplace operator in polar coordinates and the deflection function satisfying the biharmonic equation. The solution is performed by the method of initial functions on the basis of solving a symmetric problem from the concentrated effects of the jaw's teeth on the surface of the column.*

Key words: *Packer's operation, jaw's teeth, complex stress state, bending moments, transverse forces.*

1. INTRODUCTION:

During packer's operation the plunging of the jaw's teeth into the inner surface of the column, cause the damage of the column [2, 3]. Also, the complex stress state arising in the teeth adversely affects the performance of the jaw, which ultimately leads to the fracture of the jaw and to the tightness failure. Damage of the column as well as the tightness failure is extremely undesirable, since their recovery and replacement require additional work, time and costs [4].Therefore, the determination of the bending moments and transverse forces arising in the teeth of the jaw in a complex stress state has a great practical importance.

2. TASK'S FORMULATION:

During the packer landing in the production column in order to anchor it, the jaw's teeth are cut into the inner surface of the column by a certain amount [5]. Consequently, a complex stress state is created in the teeth [6] (and in the column), Wherein the bending and twisting moments M_r, M_θ,$M_{r\theta}$, transverse forces Q_r, Q_θ in the polar coordinates r, θ (Fig. 1) are determined through the deflection function according to the formulas:

$$\begin{gathered} M_r = -D\left[\frac{\partial^2 w}{\partial r^2} + \nu\left(\frac{1}{r}\cdot\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right)\right] \\ M_\theta = -D\left[\frac{1}{r}\cdot\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \nu\frac{\partial^2 w}{\partial r^2}\right] \\ M_{r\theta} = -D\left(1 - \nu\frac{\partial}{\partial r}\cdot\frac{1}{r}\cdot\frac{\partial w}{\partial r}\right) \end{gathered} \quad (1)$$

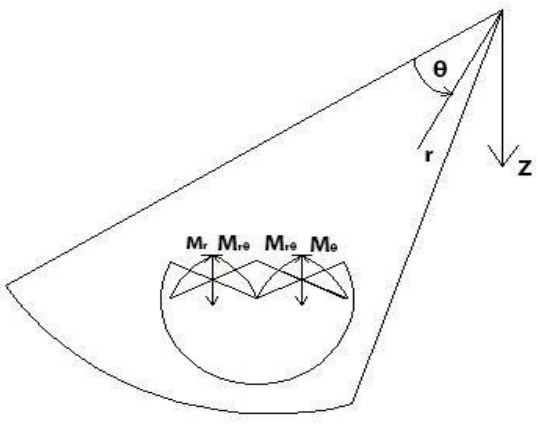

Fig. 1. Scheme of loading of the jaw's teeth by bending and twisting moments and transverse forces in polar coordinates.

Let's introduce generalized transverse forces:

$$Q_\theta = -\frac{1}{r}\cdot\frac{\partial}{\partial w}\nabla^2 w + \frac{1}{r}\cdot\frac{\partial w_{r\theta}}{\partial r}$$

$$Q_r = -D\frac{\partial}{\partial r}\nabla^2 w + \frac{1}{r}\cdot\frac{\partial w_{r\theta}}{\partial\theta} \tag{2}$$

where ∇^2 – Laplace operator in polar coordinates.

The deflection function satisfies the biharmonic equation:

$$D\nabla^2\nabla^2 w = P$$

where $P(r,\theta)$ – transverse load on the jaw's teeth.

By the method of initial functions, we obtain a representation for the basic calculated quantities: deflection W, angle of rotation $\varphi_\theta = \frac{1}{r}\cdot\frac{\partial w}{\partial\theta}$, bending moment M_θ and transverse force Q_θ for an arbitrary value of the coordinate θon the surface of the teeth through the values of the same calculated quantities, determined from its edge θ = 0, through the initial functions in the case of a homogeneous problem P [6].

In the variables t (t = lnr) and θ, the bending equation is written as an equation with constant coefficients

$$\left[(\alpha-2)^2 + \frac{\partial^2}{\partial\theta^2}\right]\left(\alpha^2 + \frac{\partial^2}{\partial\theta}\right)W = e^{4t}P \tag{3}$$

where α denotes the partial derivative with respect to t.

Let's introduce new functions instead of basic quantities W, φ_θ, $M_\theta$and Q_θ:

$$W = DW;\ \varphi = Dr\varphi_\theta, M = r^2 M_r,\ Q = r^3 Q_r \tag{4}$$

For the quantities φ, M and Q we have the ratio:

$$\varphi = \frac{\partial w}{\partial\theta}$$

$$M = -\left[\frac{\partial^2 w}{\partial\theta^2} + (1-\nu+\nu\alpha)\alpha W\right]$$

$$Q = -\frac{\partial}{\partial\theta}\left[\frac{\partial^2 w}{\partial\theta^2} + 2(1-\nu)W + (2-\nu)\alpha W - 3(1-\nu)\alpha W\right] \tag{5}$$

In accordance with the symbolic method, we write down the general solution of the homogeneous equation (3) (hereinafter, P = 0) as an ordinary differential equation, in which α is formally considered to be a constant number:

$$W = cos\alpha\theta C_1 + sin\alpha\theta C_2 + \cos(2-\alpha)\,\theta C_3 + \sin(2-\alpha)\theta C_4 \tag{6}$$

where C_1, C_2, C_3 and C_4 –formally arbitrary constants, but in fact arbitrary functions of the coordinate t.

According to the ratios (5)

$$\varphi = -\alpha sin\alpha\theta C_1 + \alpha cos\alpha\theta C_2 - (2-\alpha)\cdot\sin(2-\alpha)\theta C_3 + (2-\alpha)\cos(2-\alpha)\,\theta C_4$$

$$M = (1-\alpha)\{-(1-\nu)\alpha(cos\alpha\theta C_1 + sin\alpha\theta C_2) + [4-(1-\nu)]\cdot[\cos(2-\alpha)\,\theta C_3 + +\sin(2-\alpha)\theta C_4]\}$$

$$Q = (2-\alpha)(1-\alpha)\{(1-\nu)\alpha(sin\alpha\theta C_1 - cos\alpha\theta C_2) + [2(1+\nu)+(1-\alpha)\alpha]\alpha[-\sin(2--\alpha)]\theta C_3 + \cos(2-\alpha)\theta C_4\} \tag{7}$$

Assuming than in equations (6) and (7) $\theta = 0$ and accepting that the latter is relative to C_1, C_2, C_3 and C_4 we find dependences for them through the initial functions W_0, φ_0, $M_0$and Q_0. Substituting the results in (6) and (7), we obtain the desired solution in symbolic form by the method of initial functions:

$$W(t,\theta) = L_{ww}(\alpha,\theta)W_0(t) + L_{w\varphi}(\alpha,\theta)\varphi_0(t) + L_{wM}(\alpha,\theta)M_0(t) + L_{wQ}(\alpha,\theta)Q_0(t)$$

$$\varphi(t,\theta) = L_{w\varphi}(\alpha,\theta)W_0(t) + L_{\varphi\varphi}(\alpha,\theta)\varphi_0(t) + L_{\varphi M}(\alpha,\theta)M_0(t) + L_{\varphi Q}(\alpha,\theta)Q_0(t)$$

$$M(t,\theta) = L_{Mw}(\alpha,\theta)W_0(t) + L_{M\varphi}(\alpha,\theta)\varphi_0(t) + L_{MM}(\alpha,\theta)M_0(t) + L_{MQ}(\alpha,\theta)Q_0(t)$$

$$Q(t,\theta) = L_{Qw}(\alpha,\theta)W_0(t) + L_{Q\varphi}(\alpha,\theta)\varphi_0(t) + L_{QM}(\alpha,\theta)M_0(t) + L_{QQ}(\alpha,\theta)Q_0(t) \tag{8}$$

Differential operators of infinite high order in a closed symbolic form of a record are defined by formulas:

$$4L_{w\varphi} = \frac{1}{\alpha}\{[2(1+\nu)+(1-\nu)\alpha]sin\alpha\theta + (1-\nu)\alpha\sin(2-\alpha)\theta\}$$

$$
\begin{gathered}
4L_{wQ} = \frac{1}{\alpha(1-\alpha)(2-\alpha)}[\alpha\sin(2-\alpha)\,\theta - (2-\alpha)sin\alpha\theta] \\
4L_{wM} = \frac{1}{1-\alpha}[\cos(2-\alpha)\,\theta - cos\alpha\theta] \\
4L_{\varphi w} = -\alpha\{[4-(1-\nu)\alpha]sin\alpha\theta + (1-\nu)(2-\alpha)\cdot\sin(2-\alpha)\,\theta\} \\
4L_{ww} = [4-(1-\nu)\alpha]cos\alpha\theta + (1-\nu)\alpha\cos(2-\alpha)\,\theta \\
4L_{\varphi\varphi} = [2(1+\nu)+(1-\nu)\alpha]cos\alpha\theta + (1-\nu)(2-\alpha)\cos(2-\alpha)\,\theta \\
4L_{\varphi M} = \frac{1}{1-\alpha}[\alpha sin\alpha\theta - (2-\alpha)sin(2-\alpha)\theta] \\
4L_{\varphi Q} = \frac{1}{1-\alpha}[-\cos\alpha\,\theta + \cos(2-\alpha)\theta] \\
4L_{Mw} = (1-\nu)\alpha(\alpha-1)[4-(1-\nu)\alpha]cos\alpha\theta - \cos(2-\alpha)\,\theta] \qquad (9) \\
4L_{M\varphi} = (1-\nu)(\alpha-1)\{[2(1+\nu)+(1-\nu)\alpha] \\
4L_{MM} = \{(1-\nu)\alpha cos\alpha\theta - 1[(1-\theta)\alpha - 4]\cos(2-\alpha)\theta\} \\
4L_{MQ} = \frac{1}{2-\alpha}\{(1-\nu)(2-\alpha)sin\alpha\theta - [(1-\nu)\alpha - 4\sin(2-\alpha)\theta]\} \\
4L_{QW} = (1-\nu)\alpha(2-\alpha)(1-\alpha)\{[4-(1-\nu)]\cdot sin\alpha\theta - [(1-\nu)\alpha+2(1+\nu)]\sin(2-\alpha)\theta\} \\
4L_{Q\varphi} = (1-\nu)(2-\alpha)\,(1-\alpha)[2(1+\nu)+(1-\nu)\alpha]\cdot[-cos\alpha\theta + \cos(2-\alpha)\,\theta] \\
4L_{QM} = -(2-\alpha)\{(1-\nu)\alpha sin\alpha\theta + [(1-\nu)\alpha+2(1+\nu)]\cdot sin(2-\alpha)\theta\} \\
4L_{QQ} = (1-\nu)(2-\alpha)cos\alpha\theta + [(1-\nu)\alpha+2(1+\nu)]\cdot cos(2-\alpha)\theta
\end{gathered}
$$

In order to pass to the real solution in the form of infinite series in powers of the coordinate θ, the trigonometric functions in the equations for the operators must be expanded into power series, α must be substituted by a partial derivative with respect to t and differentiation operations under the corresponding initial functions must be performed.

3. TASK'S SOLUTION

Task's solution for jaw's teeth of an arbitrary angle $2\theta_a$ when it is loaded along one edge $\theta = \theta_a$ with arbitrary transverse q(r) and twisting m(r) loads assuming that another edge $\theta = -\theta_a$ is free from rounding and load we represent as a sum of two tasks' solutions. The first of them corresponds to the symmetrical loading of the jaw's teeth on the both sides $\theta = \pm\theta_a$ by transverse and twisting loads of intensity $\frac{1}{2}q$, $\frac{1}{2}m$ (Fig.2) and the second corresponds to the inverse-symmetric loading by the same loads (Fig. 3).

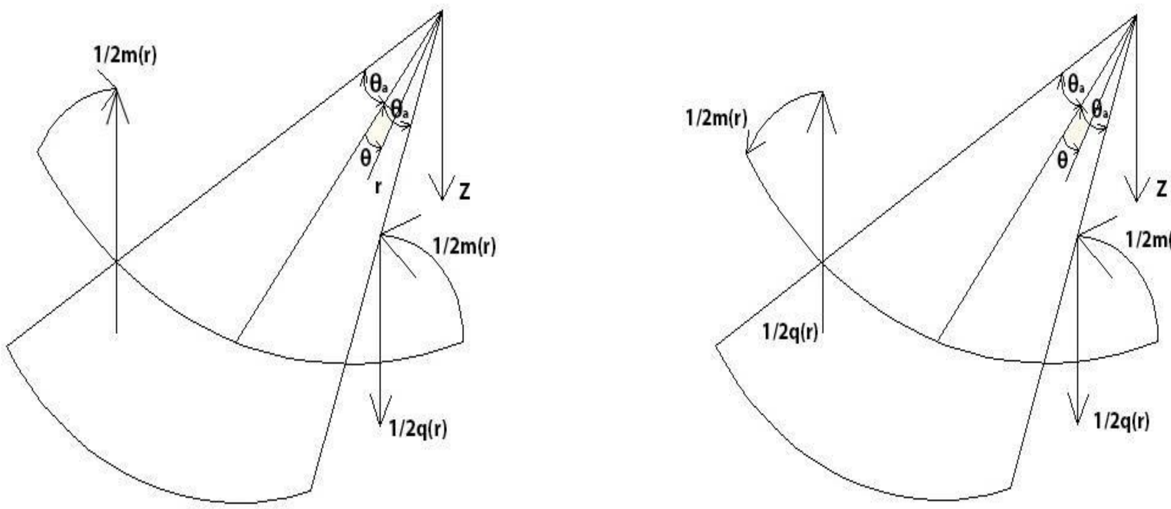

Fig.2. Calculation schemes for loading of jaw's teeth of well packer. a - symmetrical loading of the teeth by transverse and twisting loads; b - back-symmetric loading by the same loads.

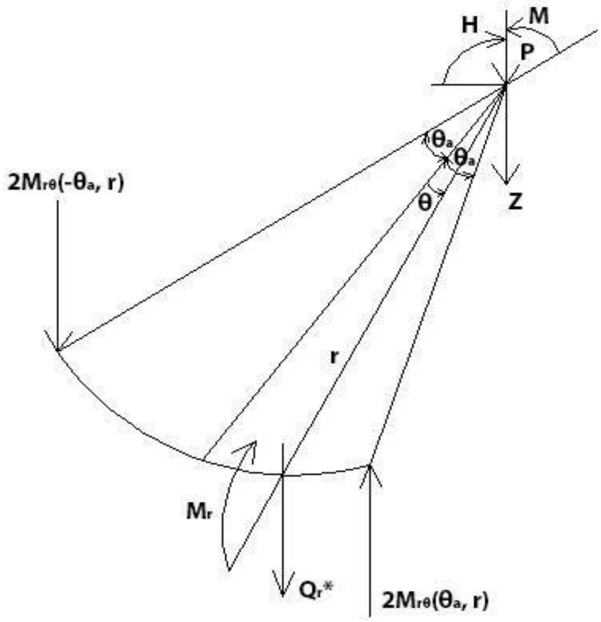

Fig.3. The calculation scheme of the equilibrium condition for the cut off part of the jaw's teeth.

4. BASIC EQUATIONS FOR THE SYMMETRIC PROBLEM

For a symmetric state at $\theta = 0$, angle of rotation φ_0 and transverse force Q_0 turn to zero. Boundary conditions on the edge $\theta = \theta_a$ are written as follows:

$$M_\theta = \frac{1}{2}m(r), \text{ at } \theta = \theta_a$$
$$Q_\theta = \frac{1}{2}q(t), \text{ at } \theta = \theta_a \tag{10}$$

Satisfying these conditions with the help of general relationships (8), we obtain a system of differential equations of infinite higher order with respect to the sought initial functions $W_0$and M_0:

$$L_{Mw}(\theta_a)W_0 + L_{ww}(\theta_a)M_0 = \frac{1}{2}e^{2t}m(t)$$
$$L_{Qw}(\theta_a)W_0 + L_{QM}(\theta_a)M_0 = \frac{1}{2}e^{3t}q(t) \tag{11}$$

where operators L are determined through formulas (9) at $\theta = \theta_a$.

We reduce the system of equations to a single equation. Assuming that the first of them is homogeneous (m = 0), we introduce the resolving function F(t):

$$W_0 = L_{MM}(\theta_a)F = \frac{1}{4}\{(1-\nu)\alpha cos\alpha\theta - [(1-\nu)\alpha - 4]\cos(2-\alpha)\,\theta_a\}F$$
$$M_0 = -L_{MW}(\theta_a)F = \frac{1-\nu}{4}\alpha(\alpha-1)[4-(1-\nu)\alpha] \times [cos\alpha\theta - \cos(2-\alpha)\,\theta_a]F \tag{12}$$

In this case, the first equation is identically satisfied, and the second will take the form:

$$[L_{Qw}(\theta_a)L_{MM}(\theta_a) - L_{QM}(\theta_a)L_{MW}(Q_a)] = \frac{1}{2}e^{3t}q(t) \tag{13}$$

Taking into account the dependences (9), we obtain

$$\frac{1-\nu}{4}\alpha(1-\alpha)(2-\alpha)[(1-\nu)(1-\alpha)sin2\theta_a + (3+\nu)sin2(1-\alpha)\theta_a]F = \frac{1}{2}e^{3t}q(t) \tag{14}$$

5. SOLVING A SYMMETRIC PROBLEM FROM CONCENTRATED EFFECTS

First, let us consider the solution of a homogeneous problem. The characteristic equation for equation (14) is transcendental. In general, at $\theta_a \neq \frac{\pi}{2}$ it contains an infinite set of complex roots. It is also satisfied by the characteristic numbers 1, 0, and 2. We call the solution determined by the last roots, elementary, since the corresponding characteristic numbers, unlike the others, do not depend on the value of the angle θ_a.

In determining the dependences for the initial functions W_0 и M_0 corresponding to the elementary solution, we proceed from the homogeneous system of equations (11) and look for them in the form:

$$W_0 = C_1 + C_2e^t + C_3te^t + C_4^*e^{2t}$$
$$M_0 = B_1 + B_2e^t + B_3te^t + B_4e^{2t} \tag{15}$$

The coefficients in these equations are unknown and are to be determined. Substituting relations (15) to equations (11) and differentiating according to formulas (9), we find the required coefficients from the condition of identical satisfaction of the equations:

$$B_1 = B_2 = B_3 = 0$$
$$C_4^* = \frac{1}{2}[(1-\nu)cos2\theta_a + 1 + \nu]C_4$$
$$B_4 = (1-\nu^2)(1 - cos2\theta_a)C_4$$

Substituting the initial functions, determined by the equations (15), into the first of the relations (8), we determine the solution for the deflection. In the initial variable r, we finally obtain:

$$w = C_1 + C_2\theta r + C_3\left(lnrcos\theta - \frac{1+\nu}{2}\theta sin\theta\right)r + \frac{1}{2}[(1+\nu)cos2\theta + (1-\nu)cos2\theta_0]C_4 4^2 \quad (16)$$

Further, using formulas (1) and (2), we find:

$$M = -D(1-\nu^2)\left[C_3\frac{cos\theta}{r} + C_4(cos2\theta + cos2\theta_a)\right]$$
$$M_0 = D(1-\nu^2)(-cos2\theta_a + cos2\theta)C_4; \quad (17)$$
$$M_{r\theta} = D(1-\nu)\left[\frac{sin\theta}{r}C_3 + (1+\nu)C_4 sin2\theta\right]$$
$$Q^* = 0$$
$$Q_r^* = 2D(1-\nu)C_3\frac{cos\theta}{r^2} + 2(1-\nu^2) \times C_4 D\frac{cos2\theta}{r} \quad (18)$$

Part of the solution with constant C_1 and C_2 corresponds to a rigid displacement of the jaw's teeth in the direction Z and its rotation with respect to the axis $\theta = \frac{\pi}{2}$. Let us show that the solution of the bending task for an unlimited number of jaw's teeth from the concentrated bending moment M applied to its vertex corresponds to the constant C_3, and the solution from the concentrated force P applied to its vertex corresponds to the constant C_4.

Let's make the equilibrium conditions for the cut off part of the jaw's teeth. Projecting all the forces on the Z-direction (Fig. 4), we obtain the first equation of equilibrium

$$\int_0^{\theta_a} \sigma_r{}^* r d\theta - 2M_{r\theta}(\theta_a, r) + \frac{P}{2} = 0 \quad (19)$$

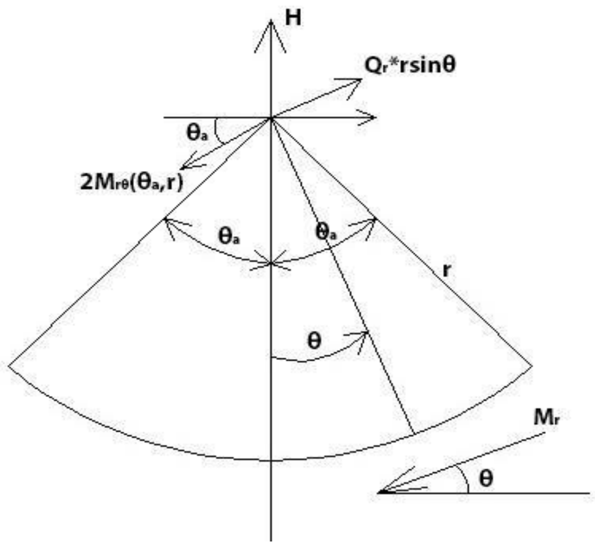

Fig. 4. The scheme of the second equilibrium equation for the sum of the moments with respect to straight line $\theta = \frac{\pi}{2}$.

Note, that in equation (19) we denote by $2M_{r\theta}(\theta_a, r)$ the moments concentrated at the corner points of the jaw's teeth with the coordinates r, θ_a, which must be added in the transition from ordinary forces to generalized ones in accordance with the theory Kirchhoff.

From the equations (18), (19) taking into account the formulas (17) we define the constants:

$$C_3 = \frac{M}{D(1-\nu)[-(3+\nu)\theta_a + \frac{1-\nu}{2} sin2\theta_a]}$$

$$C_4 = \frac{P}{2(1-\nu^2)Dsin^2\theta_a}$$

Entering the values of the constants into the equations (16) and (17), we obtain the required solution from the concentrated forces and the moment applied to the vertex of the jaw's teeth for the case of bending by the moment M:

$$W = \frac{M}{(1-\nu)} \cdot r(\ln rcos\,\theta - \frac{1+\nu}{2}\theta sin\theta)$$

$$M_r \frac{(1+\nu)Mcos\theta}{\Delta r}$$

$$M_0 = 0$$

$$M_{r\theta} = \frac{M}{\Delta} \cdot \frac{sin\theta}{r} \qquad (20)$$

where

$$\Delta = -(3+\nu)\theta_a + \frac{1-\nu}{2} sin2\theta_a$$

For the case of deflection by the force P we obtain:

$$W = \frac{Pr^2}{4(1-\nu^2)Dsin^2\theta_a}[(1+\nu)cos2\theta_a + (1-\nu) \cdot cos2\theta_a]$$

$$M_r = -\frac{P}{2sin2\theta_a}[cos2\theta + cos2\theta_a]$$

$$M_\theta = -\frac{P}{2sin2\theta_a}[cos2\theta - cos2\theta_a]$$

$$M_{r\theta} = -\frac{Psin2\theta_a}{2sin2\theta_a} \qquad (21)$$

6. CONCLUSION:

- Analytical expressions are derived for determining the bending and twisting moments, as well as the transverse forces of the jaw's teeth during the operation of packers.
- The proposed calculation methods allow us to consider solutions for a symmetric problem from concentrated effects, taking into account forces, moments and the loading of the jaw's teeth from distributed loads along the edges.

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AUTOR'S BIOGRAPHY:

1. Professor Mammadov Vasif Talib (born in 1950) is a Head of Department "Oil and Gas Equipment" in Azerbaijan State Oil and İndustry University. İn 1986 he defended dissertation work named "Development and

investigation of the elastic element of a two-pass packer in order to ensure the effect of self-packing" and received PhD degree. In 2005 he gained Doctor of Science degree with the dissertation thesis "Scientific basis of calculation and design of multi-purpose elastic element in packer structures applied in oil and gas industry". He is an author of many scientific works, patents, textbooks and manuals. He was a scientific director of many PhD graduates.

2. Mohsun Shahnazarov (born in 1981) is a Head of Department in State Agency for Safe Working in Industry and Mountain-Mine Control, Azerbaijan State Scientific-Research Institute for Labour Protection and Safety Measures. In 2006 he received a bachelor's degree in the specialty "Engineering of Technological Machines and Equipment" from Azerbaijan State Oil Academy. In 2009 he obtained master's degree from the same university and since 2017 he studies there for PhD. Theme of his doctoral thesis is "Improving of working capacity of packer's which are used during the recovery of wells with suspended activity".