Design and Structural Analysis on a Cross-Flow Turbine Runner

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Abstract: *The cross-flow turbine is used widely in mini hydro-power plants due to their simple design, easier maintenance, low initial investment and modest efficiency. Then, the cross-flow turbine has gained much attention as it is low head turbine and can be used at remote places. In this paper, the runner for 100kW cross-flow turbine is designed under net head of 15 m and flow rate of 1m³/s. The main objective of this paper is to design the model of 100kW cross-flow turbine and to study the theoretical stresses on a runner blade for two stages of it comparatively with Finite Element Analysis (FEA) results. The design parameters such as runner diameter, runner speed, runner length, turbine power, number of blades, blade pitch, radius of blade curvature and shaft diameter were calculated. The geometric modelling of runner blades have been done using Autodesk Inventor Software. The runner blade was analyzed for static case by applying the pressure load and body load. The comparison of the Von-Mises stress and displacement results for two stages with two materials are discussed. And then, the comparison of theoretical and simulation results for both stages are also described in this paper.*

Key Words: *Cross-Flow Turbine, Design Parameters, FEA, 100 kW, Runner Blade, Stress Analysis, Theoretical Stress*

1. INTRODUCTION:

In today's world, hydraulic turbines are used as a power source for a wide range of applications and electricity generation. Moreover, cost of production from water resources is relatively cheaper than that of other resources. In hydropower plants, water turbine is one of the most important parts for generating electricity. Nowadays, the Cross-flow hydraulic turbine is gaining popularity in low head and small water flow rate establishments, because of its simple structure, ease of manufacturing and straight forward installation in the site of the power plant. Cross-flow turbine is also known as Banki-Mitchell turbine [1,3].

The cross-flow turbine is composed of two major parts, the runner and the nozzle. The runner is a hollow circular section of turbine on which the curved blades are supported at the ends. The nozzle is the rectangular shape which discharges the water jet to the full width of the runner and enters the runner at an angle to the periphery of the runner. This water jet leaving the nozzle strikes the blades at first stage. Then, the water exits the first stage, crosses the runner to enter the second stage inlet and then finally passes through the runner completely. Therefore, this machine is equivalent to a double stage turbine.

2. LITERATURE REVIEW:

Many researchers have studied the static analysis of cross-flow turbine runner blades by using various software. In 2013, the design of high efficiency cross-flow turbine for hydro-power plant was proposed by Bilal Abdullah Nasir. The researcher has described the complete design procedure of cross-flow turbine based on the theory of cross-flow turbine. To obtain a cross-flow turbine with maximum efficiency, the turbine parameters must be included in the design such as runner diameter, runner length, runner speed, turbine power, water jet thickness, blade spacing, number of blades, radius of blades, attack angle and blade inlet and exit angle. The results showed that the maximum efficiency was found to be 88 percent constant for different values of head and flow rate [1].

Mrudang and Nirav Oza are studied on "Design and Analysis of High Efficiency Cross-Flow Turbine for Hydro-Power Plant" in 2016. In this paper, they are designed a cross-flow turbine with maximum efficiency. The turbine blades are analysed under a head of 10m and flow rate of 0.315 m^3/s. Hydraulic pressure load on blades is calculated and applied on 6 blades in ANSYS 16 after being modelling in Solid Works software [2].

Mockmore C. A. and Merryfield F. have developed the Banki water turbine theory. The Banki turbine is an atmospheric radial flow wheel which derives its power from kinetic energy of the water jet. Calculations theory for each specifications of the cross-flow turbine were presented. Model of cross-flow turbine was developed and compared with test nozzle and simplified nozzle. The result showed that the Banki turbine can be operated efficiently on a wider range of openings than most turbines, maximum efficiency occurs at practically a constant speed for all gate openings at constant head. The advantages are simplicity and economy of construction [3].

Verharrt P. have developed blade design calculations for water turbines of the Banki type. The methods for calculating the turbine blades specifications are explained. Calculations of constructive dimensions, fluid dynamical dimensions, torque transmitted by a blade channel, point of application of hydraulic loads, distributed load on a blade, bearing stress on blade section and strength of products were presented for all the main dimensions of a Banki turbine blade design [4].

In the current paper, a complete design of 100 kW cross-flow turbine is calculated. And then, theoretical stresses on a runner blade of 100 kW cross-flow turbine runner are carried out for two stages and stress analysis on a runner blade is done with two materials by using Autodesk Inventor Software.

3. DESIGN PROCEDURE OF CROSS-FLOW TURBINE RUNNER:

The following equations are necessary to calculate the design of 100 kW Cross-Flow turbine. The principle dimensions of a Cross-Flow turbine are considered by the runner diameter the length of the runner. The main parameters of Cross-Flow turbine blade such as radius of blade curvature, number of blades and blade pitch [1,3,8,9].

Turbine input power,	$P = \dfrac{P_G}{0.94}$ (3.1)
Specific speed,	$N_s = \dfrac{172.556}{H^{0.425}}$ (3.2)
Rotation speed,	$N = \dfrac{N_S \times H^{5/4}}{\sqrt{P}}$ (3.3)
Outer diameter of runner,	$D_1 = \dfrac{k_1 \times 60 \times C_V \sqrt{2gH}}{\pi N}$ (3.4)
Inner diameter of runner,	$D_2 = \dfrac{2}{3} D_1$ (3.5)
Pitch circle diameter	$D_o = 0.74\, D_1$ (3.6)
Runner length,	$L = \dfrac{Q}{k\, D_1\, C_V \sqrt{2gH}}$ (3.7)
Radius of blade curvature,	$r = 0.16\, D_1$ (3.8)
Radial rim width,	$a = R_1 - R_2$ (3.9)
Inlet blade angle	$\beta_1 = \tan^{-1}(2\tan \alpha_1)$ (3.10)
Central angle of blade	$\tan \dfrac{\delta}{2} = \dfrac{\cos\beta_1}{\sin\beta_1 + \dfrac{R_2}{R_1}}$ (3.11)
Nozzle throat width	$S_1 = k\, D_1$ (3.12)
Width of nozzle	$b = 0.95B$ (3.13)
Number of blade	$n = \dfrac{\pi D_1}{t}$ (3.14)
Blade pitch	$p_1 p_2 = \dfrac{2\pi R_o}{n}$ (3.15)
Shaft diameter	$d_s = \sqrt[3]{\dfrac{16T}{\pi\tau}}$ (3.16)

4. GEOMETRIC MODELLING OF CROSS-FLOW TURBINE:

The designs of Cross-flow turbine runner and blade are performed according to the calculated results. They are 3D modelling of cross-flow turbine runner and single blade with a blade section length. The Table 4.1 shows the theoretically calculated data at net head is 15 m and flow rate is 1 m^3/s.

Table 4.1. SPECIFICATIONS OF CROSS-FLOW TURBINE

No.	Description	Symbol	Unit	Result
1	Turbine Output Power	P_T	kW	106
2	Turbine Speed	N	rpm	157
3	Outer Diameter of Runner	D_1	mm	800
4	Inner Diameter of Runner	D_2	mm	534
5	Pitch Diameter of Runner	D_o	mm	592
6	Length of Runner	L	mm	1100
7	Radius of Blade Curvature	r	mm	128
8	Radial Rim Width	a	mm	133
9	Inlet Blade Angle	β_1	degree	30
10	Central Angle of Blade	δ	degree	73
11	Blade pitch	p_1p_2	mm	103
12	Number of Blade	n	blade	18
13	Nozzle Throat Width	S_1	mm	70
14	Shaft diameter	d_s	mm	110

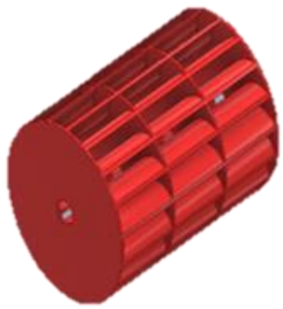


Fig.4.1 3D Modelling of Cross-Flow Turbine Runner

Fig.4.2 One Blade with a Blade Section Length

5. THEORETICAL STRESS ON RUNNER BLADE OF 100kW CROSS-FLOW TURBINE:

The calculated result data are used to calculate the theoretical stress on runner blade for two stages. In this calculation, the blades are treated as uniform beams of constant cross section, fixed at both ends and the hydraulic force of the water jet is treated as a uniformly distributed load along the entire length of the blades are assumed [4].

5.1 Length of a Blade Section

Thickness of the supporting board for each is assumed as 0.0127 m. In this design, there are 4 supporting boards and the length of runner is divided into 3 sections. Therefore; the length of a blade section between supports l is 0.35 m.

5.2 Torque Transmitted by a Blade Channel

The torque transmitted by a blade channel T_C can be calculated as follow [5]:

For First stage,

$$T_{c(1\text{-}2)} = Q\,\rho\,(v_1 R_1 \cos\alpha_1 - u_2 R_2) \qquad (5.1)$$

For Second stage,

$$T_{c(3-4)} = Q\,\rho\,u_3 R_2 \qquad (5.2)$$

5.3 Point of Application of the Hydraulic Forces

The position of the blade in relation to the tangential force F can be shown Fig.5.1.

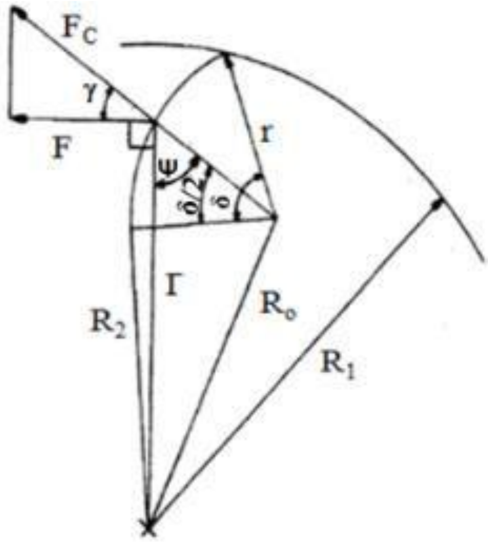


Fig.5.1 Forces on a blade [4]

In the triangle formed by Γ, R_o and r, the centric force F_c works along r while the hydraulic force F is perpendicular to Γ.Therefore, the angle γ is the complement of the included angle ψ between Γ and r [4].

The distance from the middle of the blade curvature to the centre of the runner Γ can be given as:

$$\Gamma = 2.6236\, r \quad (5.3)$$

Using the cosine rule;

$$R_o^2 = \Gamma^2 + r^2 - 2\Gamma r \cos\psi \quad (5.4)$$

Included angle, $$\psi = \cos^{-1}\left(\frac{\Gamma^2 + r^2 - R_o^2}{2\Gamma r}\right) \quad (5.5)$$

The complementary angle, $$\gamma = \frac{\pi}{2} - \psi \quad (5.6)$$

The force F is the projection of the centric force F_c. Therefore, the centric force can be calculated as follow:

$$F_c = \frac{F}{\cos\gamma} \quad (5.7)$$

The force F can be found from dividing the torque T_c by Γ,

$$F = \frac{T_c}{\Gamma} \quad (5.8)$$

5.4 *D*istributed *L*oad and *H*ydraulic *P*ressure on *T*urbine *B*lade

The distributed load can be obtained from dividing the centric force F_c by the section length l [4]:

$$W = \frac{F_c}{l} \quad (5.9)$$

The hydraulic pressure on a blade can also be calculated by using the following equation.

Therefore, the area of the concave side on turbine blade surface A can be found as follow:

$$A = r\,\delta\, l \quad (5.10)$$

$$P = \frac{F_c}{A} \quad (5.11)$$

5.5 Bending Moment and Bending Stress on Blade Section

The maximum bending moment states that it is directly proportional to the distributed load and to the square of the length of the beam as below [4]:

$$M = \frac{Wl^2}{12} \quad (5.12)$$

A further relation exists between the bending moment and the bending stress as follow:

$$\sigma = \frac{M\,e}{I_x} \tag{5.13}$$

Where, the extreme fibre distance from the neutral plane e is given as:

$$e = \frac{10^{-4}\,(2631r^2 + 10644rt_o + 6219t_o{}^2)}{(2r + t_o)} \tag{5.14}$$

Where, the area moment of inertia I_x can be carried out as follow:

$$I_x = \frac{10^{-4}\,(91\,r^4 t + 181\,r^3 t^2 + 2019\,r^2 t^3 + 1928\,rt^4 + 321\,t^5)}{(2r + t)} \tag{5.15}$$

TABLE 5.1. THEORETICAL RESULTS ON A BLADE CHANNEL FOR TWO STAGES

No.	Description	Symbols	Unit	Result
1.	Torque	$T_{c(1-2)}$, $T_{c(3-4)}$	N-m	4708, 1359
2.	Distance from the middle of the blade curvature to the centre of the runner	Γ	m	0.3358
3.	Included angle	ψ	degree	60
4.	Complementary angle	γ	degree	30
5.	Tangential Force	$F_{(1-2)}$, $F_{(3-4)}$	N	14020, 4047
6.	Centric Force	$F_{c(1-2)}$, $F_{c(3-4)}$	N	16189, 4673
7.	Distributed Load	$W_{(1-2)}$, $W_{(3-4)}$	N/m	46254, 13351
8.	Hydraulic Pressure	$P_{(1-2)}$, $P_{(3-4)}$	MPa	0.284, 0.082
9.	Bending Moment	$M_{(1-2)}$, $M_{(3-4)}$	N-m	472,136
10.	Area Moment of Inertia	I_x	m^4	1.195×10^{-7}
11.	Bending stress	$\sigma_{(1-2)}$, $\sigma_{(3-4)}$	MPa	85, 25

6. IMPORTANCE OF FINITE ELEMENT ANALYSIS:

Finite Element Analysis (FEA) is a numerical procedure that can be used to obtain solutions to a large class of engineering problems such as stress analysis, heat transfer, electromagnetism and fluid flow. The finite element stress analysis method is employed to determine whether the strength of a component (part) or set of components is sufficient when subjected to a given set of applied loads, material and boundary conditions. Inventor software is based on the FEA method to obtain the stress analysis result. There are two types of stress analysis in Autodesk Inventor Professional. They are statics analysis and modal analysis [6].

7. PERFORMANCE OF STATIC ANALYSIS:

Static analysis is an engineering discipline that determines the stress in materials and structures subject to static or dynamic forces or loads. The aim of the analysis is usually to determine whether the element or collection of elements, usually referred to as a structure or component, can safely withstand the specified forces and loads. This is achieved when the determined stress from the applied force is less than the yield strength the material is known to be to withstand. This stress relationship is commonly referred to as factor of safely (FOS) and is used in many analyses an indicator of success or failure in analysis [6].

$$\text{Factor of Safety} = \frac{\text{Yield Stress}}{\text{Calculated Stress}} = \frac{\text{Ultimate Stress}}{\text{Calculated Stress}}$$

8. STRESS ANALYSIS WORKFLOW

The following Fig.8.1 describes the procedure for static structural analysis by using Autodesk Inventor.

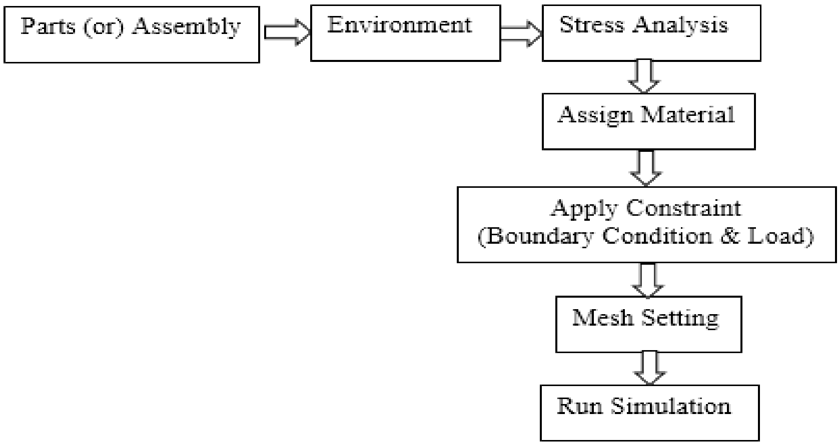


Fig.8.1 Flow chart for stress analysis

9. STRESS ANALYSIS OF CROSS-FLOW TURBINE RUNNER BLADE:

The first step of analysis was to assign the material. The properties for low alloy steel and cast iron are shown as following table [7]:

TABLE 9.1. PROPERTIES FOR LOW ALLOY STEEL AND CAST IRON

Material	Yield Strength	Ultimate Tensile Strength	Young's Modulus	Density	Poisson's Ratio
Low Alloy Steel	250 MPa	400 MPa	207 GPa	7.85 g/cm^3	0.3
Cast Iron	276 MPa	414 MPa	169 GPa	7.10 g/cm^3	0.29

The second step was given the fixed and frictionless constraints under boundary condition on a blade section.

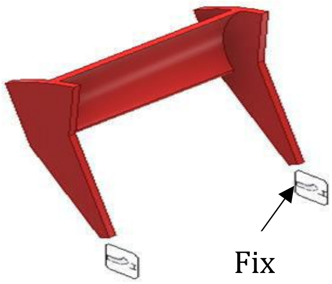


Fig.9.1 Fixed constraint

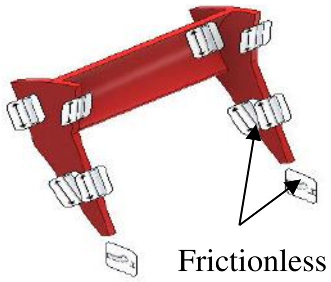


Fig.9.2 Frictionless constraint

The next step was applied on the turbine blade with pressure load and body load.

TABLE 9.2. PRESSURE LOAD FOR TWO STAGES

Load Type	Pressure (first stage)	Pressure (second stage)
Magnitude	0.284 MPa	0.082 MPa

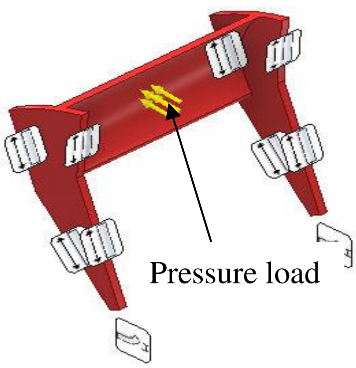


Fig.9.3 Pressure load on blade

TABLE 9.3. BODY LOAD FOR TWO STAGES

Load Type	Body load
Magnitude	942 deg/s

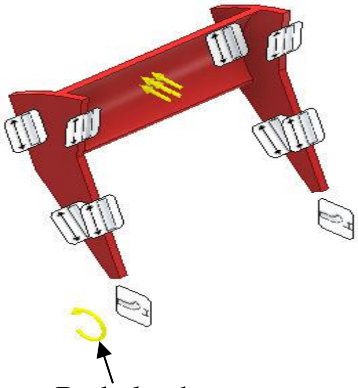
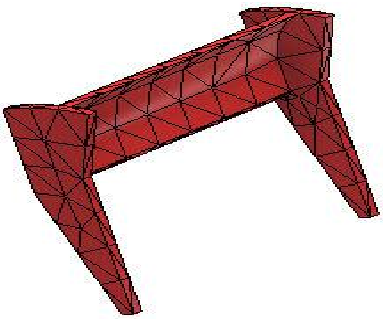


Fig.9.4 Body load on blade

The next point was to do the mesh setting which is generated by using Autodesk Inventor software. A complete workflow of the meshing is presented in Fig.3.

TABLE 9.4. MESH SETTING ON TURBINE BLADE

Average Element Size (fraction of model diameter)	0.15
Minimum Element Size (fraction of average size)	0.2
Grading Factor	2.0
Maximum Turn Angle	60^0
Create Curved Mesh Elements	Yes



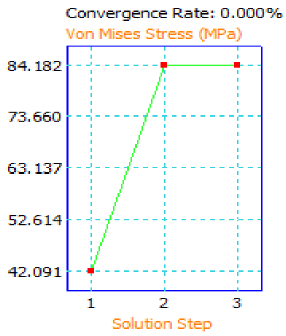


Fig.9.5 Mesh Setting and Convergence plot

10. SIMULATION RESULT WITH TWO MATERIALS FOR TWO STAGES:

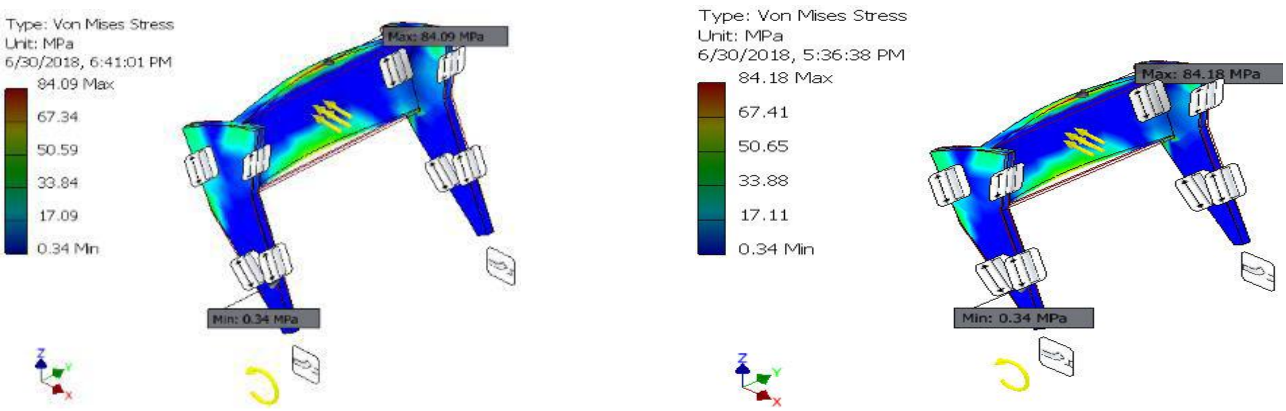


Fig.10.1 Von Mises stress for first stage by using low alloy steel and cast iron

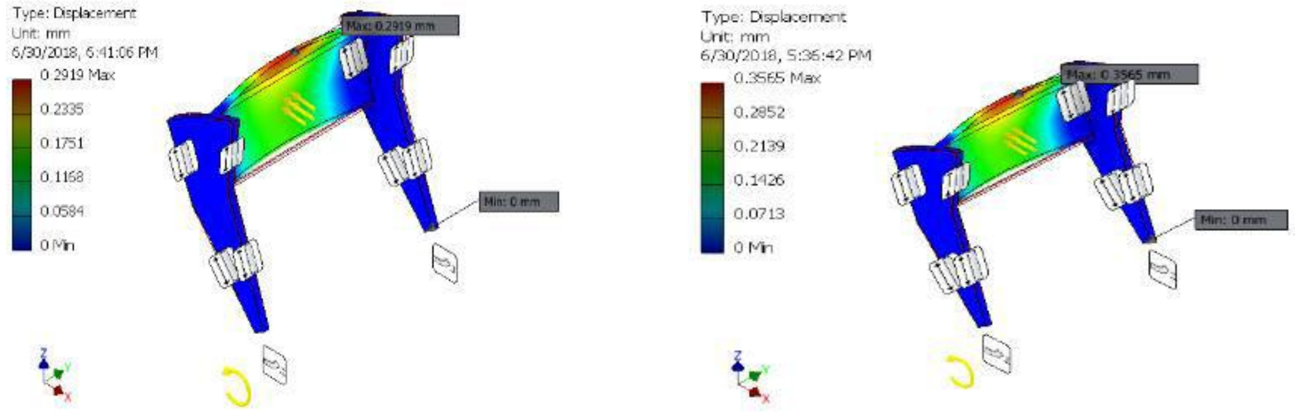


Fig.10.2 Displacement for first stage by using low alloy steel and cast iron

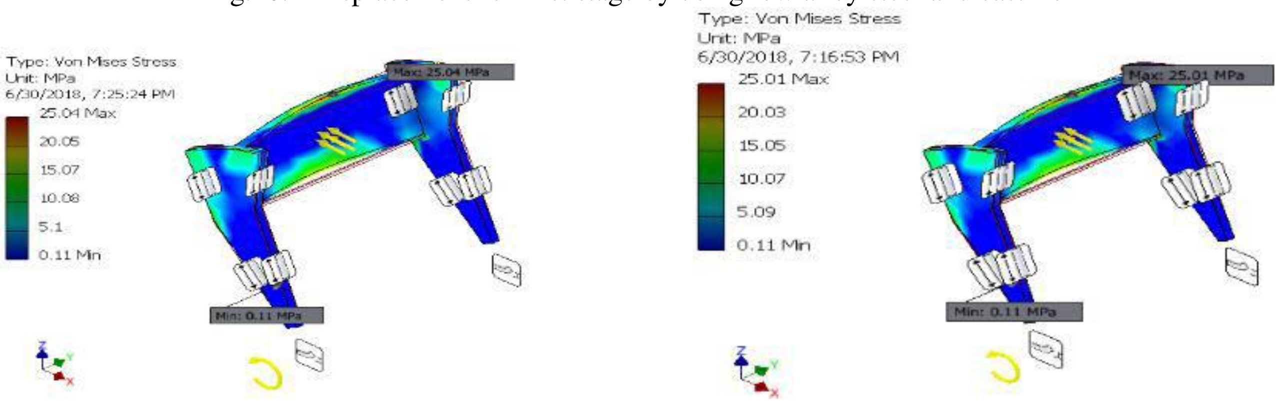


Fig.10.3 Von Mises stress for second stage by using low alloy steel and cast iron

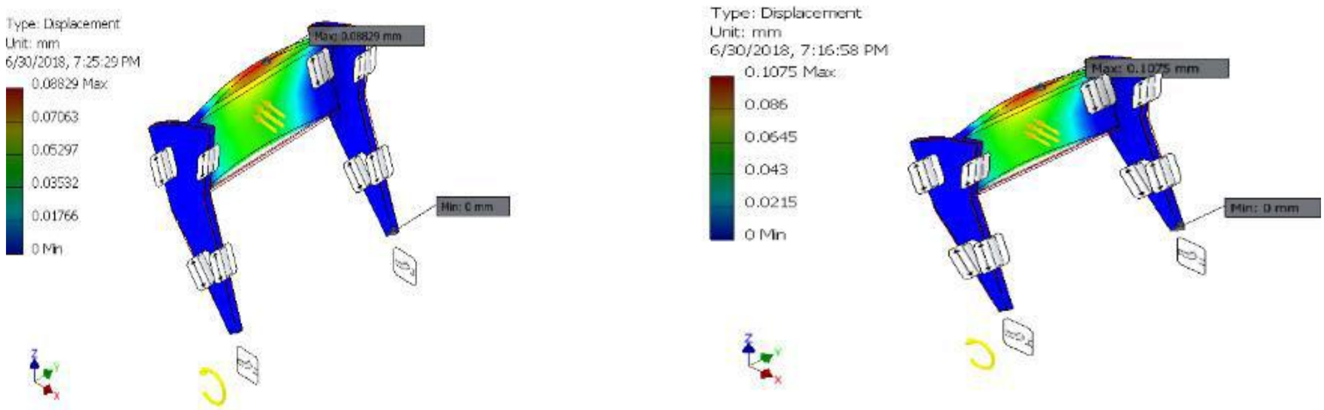


Fig.10.4 Displacement for second stage by using low alloy steel and cast iron

11. COMPARISON OF SIMULATION RESULT FOR VON-MISES STRESS AND DISPLACEMENT WITH TWO MATERIALS:

TABLE 11.1. COMPARISON SIMULATION RESULTS OF VON-MISES STRESS AND DISPLACEMENT WITH TWO MATERIALS

Stages	First Stage		Second Stage	
Material	Low Alloy Steel	Cast Iron	Low Alloy Steel	Cast Iron
Von-Mises Stress (MPa)	84.09	84.18	25.04	25.01
Displacement (mm)	0.292	0.357	0.088	0.108

The computed results for static structural analysis with two types of materials are compared with Von-Mises stress and displacement. The two materials have nearly the same Von-Mises stress on the blade. The displacement of the blade with low alloy steel is lower than that of cast iron. So, low alloy steel is suitable material for this research. Moreover, low alloy steel is better corrosion resistance than cast iron. It is also easier to fabricate, machine and manufacture than the cast iron one.

12. COMPARISON OF THEORETICAL AND SIMULATION RESULTS FOR TWO STAGES:

TABLE 12.1. COMPARISON OF THEORETICAL STRESS AND ANALYSIS STRESS FOR TWO STAGES

Stress (stage)	Theoretical Stress (MPa)	Simulation Stress (MPa)	Difference Percentage (%)
Von-Mises Stress (first stage)	85	84	1
Von-Mises Stress (second stage)	25	25	0

This table shows the comparison of theoretical and simulation stresses for both stages. In this design, using low alloy steel is safe as the maximum von-Mises stress (85 MPa) is well below its yield strength (250 MPa).

13. CONCLUSION:

In this paper, the design procedure for 100 kW cross-flow turbine has been presented. Then, the parameters for cross-flow turbine are calculated under head (15 m) and discharge (1 m^3/s) to generate 100kW. The diameter of runner is 800mm, length is 1100 mm and number of blade is 18 blades. The turbine speed is 157 rpm, blade pitch is 103 mm and central angle of blade is 73°. The diameter of the shaft is 110 mm and nozzle throat width is 70 mm. And then, static analysis for low alloy steel and cast iron reveals that the maximum Von-Mises stresses on blade for first stage are 84.09 MPa and 84.18 MPa. For second stage, those values are 25.04 MPa and 25.01 MPa respectively. The maximum displacements on runner blade for first stage are 0.292 mm and 0.357 mm and that for second stage are 0.088 mm and 0.108 mm respectively. According to the resulted data, low alloy steel will be used because its displacement is lower than that of cast iron. After that, the theoretical and simulation stresses for both stages are also compared. In this comparison, the difference percentage of Von-Mises stress for first stage is 1%. In the second stage, there is no difference percentage. Based on above findings, it can be concluded that the simulation results of the stresses acting on the blade are quite close to those of theoretical calculations. The factor of safety for low alloy steel is 3. Therefore, low alloy steel sheet can be selected as a suitable material for runner blades in this research.

REFERENCES:

1. Bilal Abdullah Nasir. “Design of High Efficiency Cross-Flow Turbine for Hydro-Power Plant”, International Journal of Engineering and Advanced Technology (IJEAT), Volume-2, Issue-3, February 2013.
2. Mrudang and Nirav Oza. “ Design and Analysis of High Efficiency Cross-Flow Turbine for Hydro-Power Plant”, International Journal of Engineering and Advanced Technology (IJEAT), ISSN: 2249 – 8958, Volume-5, Issue-4, April 2016.
3. C.A.Mockmoreand Fred Merryfield., February, 1949. "The Banki water turbine", Engineering Experiment Station Bulletin Series, No. 25.
4. Verhaart. P. March 1983. “Blade Calculations for Water Turbines of the Banki Type”. Technical report, Eindhoven University of Technology.
5. Martin Andreas Just, Gard Sviggum Saabye June10,2013. “Assessment of Structural Integrity and Potential Improvements of Micro Hydropower Turbine”.
6. Wasim Younis 2014. Up and Running with Autodesk Inventor, PART1- Stress and Frame Analysis.
7. William D. Callister, Jr. and David G. Rethwisch 1940. Materials Science and Engineering an Introduction, 8thEdition.
8. Dr. Franzke, From Lecture, Voith Siemen T.M. Co. Ltd. Germany.
9. R.S. KHURMI and J.K. GUPTA 2005. “A Textbook of Machine Design”. For the students of U.P.S.C. (Engg. Services); B.Sc. Engg; Section ‘B’ of A.M.I.E. (India)and Diploma Courses; EURASIA PUBLISHIHG HOUSE (Pvt.) LTD. RAM NAGAR, NEW DELHI-110 055.